IIT-JAM PHYSICS [2012 PAPER]

Instructions:

• Attempt all the 25 questions.

• Questions 1-15 (Objective questions) carry *six* marks each and each questions 16-25 (Subjective questions) carry *twenty one* marks each.

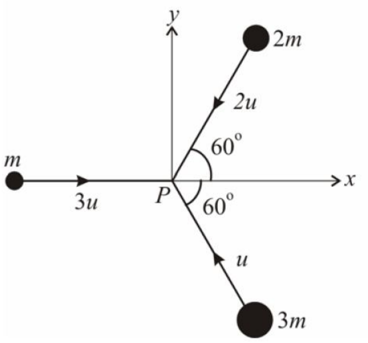
1. Given a function $f(x,t)$ of both position x and time t, the value of $\frac{\partial \dot{f}}{\partial \dot{x}}$ (where $\dot{f} = \frac{df(x,t)}{dt}$, $\dot{x} = \frac{dx}{dt}$) is:

(a) $\frac{\partial^2 f}{\partial x^2}$ (b) $\frac{\partial f}{\partial x}$ (c) $\frac{\dot{f}}{\dot{x}}$ (d) $\frac{df}{dx}$

2. If $\vec{F}$ is a constant vector and $\vec{r}$ is the position vector then $\vec{\nabla}(\vec{F}.\vec{r})$ would be

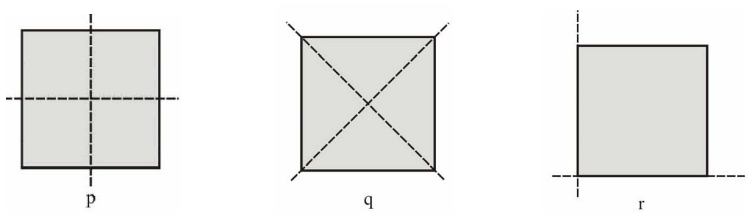
(a) $(\vec{\nabla}.\vec{r})\vec{F}$ (b) $\vec{F}$ (c) $(\vec{\nabla}.\vec{F})\vec{r}$ (d) $|\vec{r}|\vec{F}$

3. Three masses m, 2m and 3m are moving in x-y plane with speeds 3u, 2u and u, respectively, as shown in the figure. The three masses collide at the same time at P and stick together. The velocity of the resulting mass would be



(a) $\frac{u}{12}(\hat{x}+\sqrt{3}\hat{y})$ (b) $\frac{u}{12}(\hat{x}-\sqrt{3}\hat{y})$ (c) $\frac{u}{12}(-\hat{x}+\sqrt{3}\hat{y})$ (d) $\frac{u}{12}(-\hat{x}-\sqrt{3}\hat{y})$

4. The figure shows a thin square sheet of metal of uniform density along with possible choices for a set of principal axes (indicated by dashed lines) of the moment of inertia, lying in the plane of the sheet. The correct choice(s) for the principal axes would be

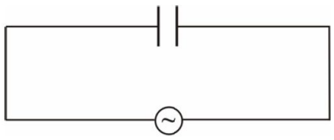


(a) p, q and r (b) p and r (c) p and q (d) p only

5. A lightly damped harmonic oscillator loses energy at the rate of 1% per minute. The decrease in amplitude of the oscillator per minute will be closest to

(a) 0.5% (b) 1% (c) 1.5% (d) 2%

6. A parallel plate air-gap capacitor is made up of two plates of area 10 cm^2 each kept at a distance of 0.88 mm. A sine wave of amplitude of 10V and frequency 50 Hz is applied across the capacitor as shown in the figure. The amplitude of the displacement current density (in mA/m^2) between the plates will be closest to



(a) 0.03 (b) 0.30 (c) 3.00 (d) 30.00

7. A tiny particle of mass 1.4×10^{-11} kg is floating in air at 300K. Ignoring gravity, its r.m.s. speed $(\text{in } \mu\text{m/s})$ due to random collisions with air molecules will be closest to
(a) 0.3 (b) 3 (c) 30 (d) 300

8. When the temperature of a blackbody is doubled, the maximum value of its spectral energy density, with respect to that at initial temperature, would become
(a) 1/16 times (b) 8 times (c) 16 times (d) 32 times

9. Light takes 4 hours to cover the distance from Sun to Neptune. If you travel in a spaceship at a speed 0.99c (where 'c' is the speed of light in vacuum), the time (in minutes) required to cover the same distance measured with a clock on the spaceship will be approximately
(a) 34 (b) 56 (c) 85 (d) 144

10. ${}^{60}_{27}Co$ is a radioactive nucleus of half-life $2\ell n\, 2\times10^{8}$ s. The activity of 10 g of ${}^{60}_{27}Co$ in disintegration per second is:

(a) $\frac{1}{5}\times10^{10}$ (b) 5×10^{10} (c) $\frac{1}{5}\times10^{14}$ (d) 5×10^{14}

11. An X-ray beam of wavelength 1.54Å is diffracted from the (110) planes of a solid with a cubic lattice of lattice constant 3.08Å. The first-order Bragg diffraction occurs at

(a) $\sin^{-1}\left(\frac{1}{4}\right)$ (b) $\sin^{-1}\left(\frac{1}{2\sqrt{2}}\right)$ (c) $\sin^{-1}\left(\frac{1}{2}\right)$ (d) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

12. The Boolean expression $P+\overline{P}Q$, where P and Q are the inputs to a circuit, represents the following logic gate
(a) AND (b) NAND (c) NOT (d) OR

13. **Group-I** contains x- and y-components of the electric field and **Group-II** contains the type of polarization of light.

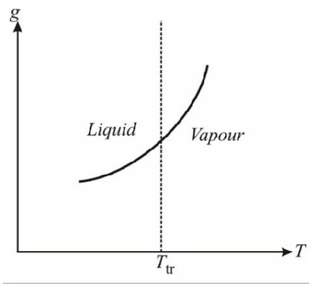
Group-I	**Group-II**
P. $E_x=\frac{E_0}{\sqrt{2}}\cos(\omega t+kz)$, $E_y=E_0\sin(\omega t+kz)$	1. Linearly polarized
Q. $E_x=E_0\sin(\omega t+kz)$, $E_y=E_0\cos(\omega t+kz)$	2. Circularly polarized
R. $E_x=E_1\sin(\omega t+kz)$, $E_y=E_2\sin(\omega t+kz)$	3. Unpolarized
S. $E_x=E_0\sin(\omega t+kz)$, $E_y=E_0\sin\left(\omega t+kz+\frac{\pi}{4}\right)$	4. Elliptically polarized

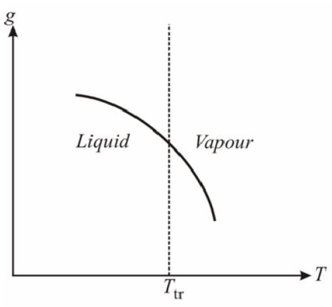
The correct set of matches is

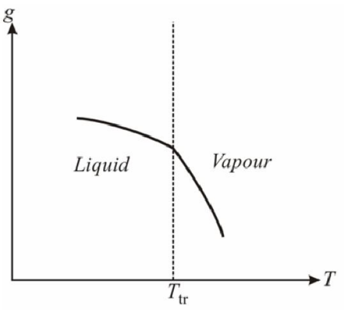
(a) $P \rightarrow 4;\ Q \rightarrow 2;\ R \rightarrow 4;\ S \rightarrow 1$ (b) $P \rightarrow 1;\ Q \rightarrow 3;\ R \rightarrow 1;\ S \rightarrow 4$

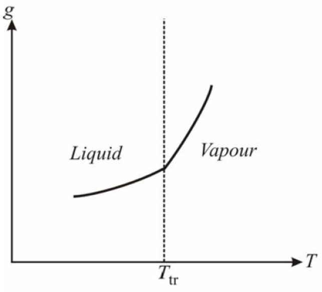
(c) $P \rightarrow 4;\ Q \rightarrow 2;\ R \rightarrow 1;\ S \rightarrow 4$ (d) $P \rightarrow 3;\ Q \rightarrow 1;\ R \rightarrow 3;\ S \rightarrow 2$

14. For a liquid to vapour phase transition at T_{tr}, which of the plots between specific Gibbs free energy 'g' and temperature 'T' is correct?

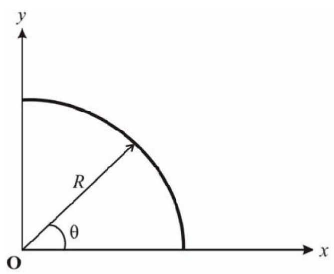
(a) 

(b) 

(c) 

(d) 

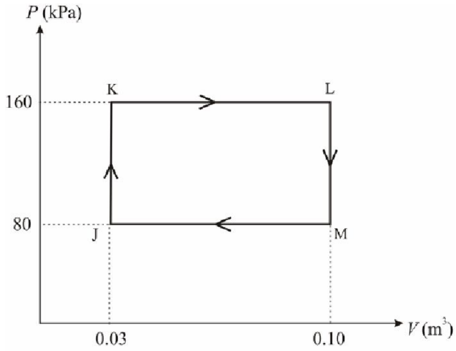
15. A segment of a circular wire of radius R, extending from $\theta = 0$ to $\pi/2$, carries a constant linear charge density, λ. The electric field at origin 'O' is:



(a) $\frac{\lambda}{4\pi\varepsilon_0 R}(-\hat{x}-\hat{y})$ (b) $\frac{\lambda}{4\pi\varepsilon_0 R}\left(-\frac{1}{\sqrt{2}}\hat{x}-\frac{1}{\sqrt{2}}\hat{y}\right)$

(c) $\frac{\lambda}{4\pi\varepsilon_0 R}\left(-\frac{1}{2}\hat{x}-\frac{1}{2}\hat{y}\right)$ (d) 0

16. The P-V diagram below represents an ideal monatomic gas cycle for 1 mole of a gas. In terms of the gas constant R. Calculate the temperatures at the points J, K, L and M. Also calculate the heat rejected and heat abosrbed during the cycle, and the efficiency of the cycle. **[Marks-21]**



17. 2 kg of a liquid (specific heat = 2000 J K^{-1} kg^{-1}, independent of temperature) is heated from 200 K to 400 K by either of the following two processes P_1 and P_2.

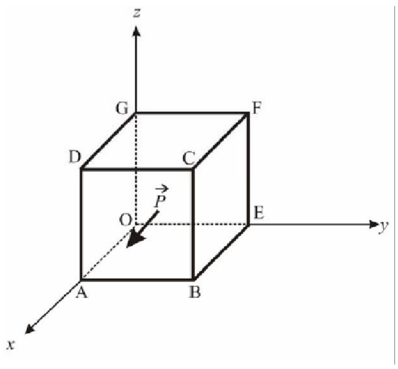
P_1: bringing it in contact with a reservoir at 400 K.

P_2: bringing it first in contact with a reservoir at 300 K till equilibrium is reached, and then bringing it in contact with another reservoir at 400K.

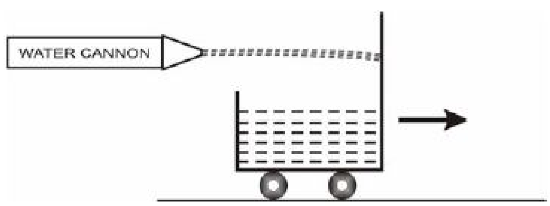
Calculate the change in the entropy of the liquid and that of the universe in processes P_1 and P_2. Neglect any change in volume of the liquid. **[Marks-21]**

18. (a) Two concentric, conducting spherical shells of radii R_1 and R_2 ($R_1 < R_2$) are maintained at potentials V_1 and V_2, respectively. Find the potential and electric field in the region $R_1 < r < R_2$. **[Marks-12]**

(b) A polarized dieelectric cube of side l is kept on the x-y plane as shown. If the polarization in the cube is $\vec{P} = kx\hat{x}$, where 'k' is a positive constant, then find all the bound surface charge densities and volume charge density **[Marks-09]**

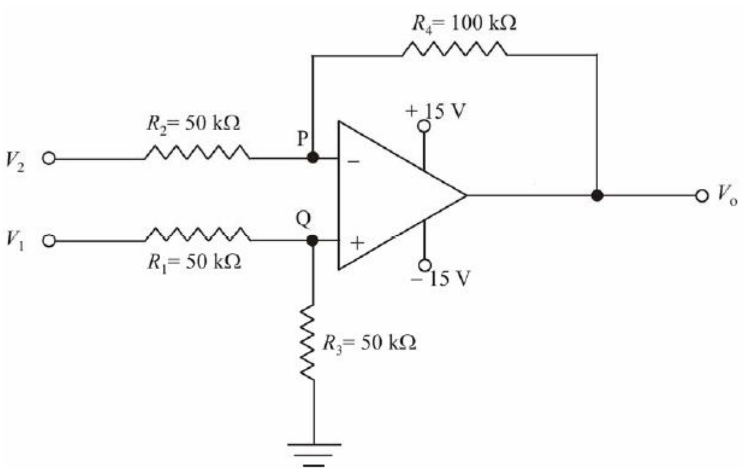


19. A water cannon starts shooting a jet of water horizontally, at r = 0, into a heavy trolley of mass 'M" placed on a horizontal ground . The nozzle diameter of the water cannon is d, the density of water is ρ , and the speed of water coming out of the nozzle is u. Find the speed of the trolley as a function of time. Assume that all the water from the jet is collected in the trolley. Neglect all frictional losses. **[Marks-21]**



20. A long straight solenoid of radius R and n truns per unit length carries a current $I = \alpha t$, where α is a constant, 't' is time and remains finite. The axis of the solenoid is along the z-axis. Find the magnetic field, electric field and the Poynting vector inside the solenoid. Show these vectors at some instant t_1 at any point (i) on the axis of the solenoid, and (ii) at a distance r (< R) from the axis. **[Marks-21]**

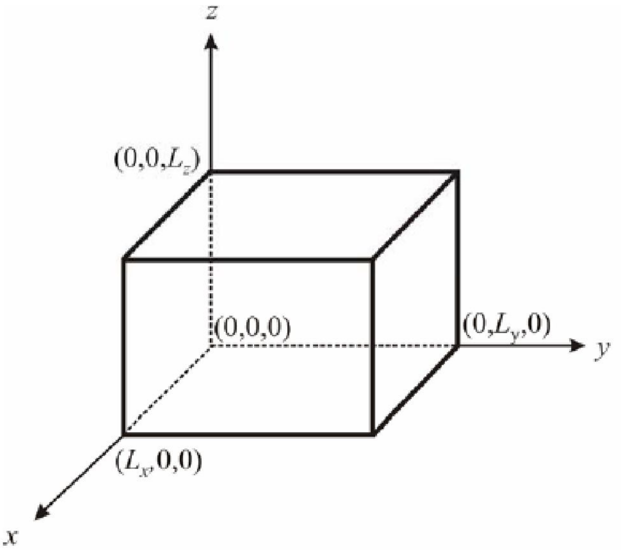
21. In the operational amplifier circuit shown below, input voltage, $V_1 = \frac{2}{3}V$ and $V_2 = \frac{1}{2}V$ are applied.



(a) Determine the current flowing through a resistance R_4 and the output voltage V_0. **[Marks-12]**

(b) In the above circuit, if V_1 is grounded and square pulses of peak voltage 1V and frequency 100 Hz are applied at V_2, determine the voltage and phase change of the output pulses. **[Marks-09]**

22. A particle of mass 'm' is confined in a potential box of sides L_x, L_y and L_z, as shown in the figure. By solving the Schrödinger equation of the particle, find its eigenfunctions and energy eigenvalues. **[Marks-21]**



23. A particle of mass 'm' and charge 'q' moves in the presence of a time-independent magnetic field $\vec{B}(\vec{r})$. Set up Newton's equation of motion for the particle.

Since for a magnetic field $\vec{\nabla}.\vec{B} = 0$, one can write $\vec{B} = \vec{\nabla} \times \vec{A}$, where $\vec{A}$ is a function of position. Calcualte $\frac{d\vec{A}}{dt}$ as seen by the moving particle. Show that $\frac{d}{dt}\left(\vec{p} + q\vec{A}\right)$, where $\vec{p}$ is the momentum of the particle, can be written as 'q' times the gradient of a function. **[Marks-21]**

24. Consider a periodic function f(x), with periodicity 2π

$$f(x) = \begin{cases} c & 0 \le x < \pi \\ 0 & \pi \le x < 2\pi \end{cases}$$

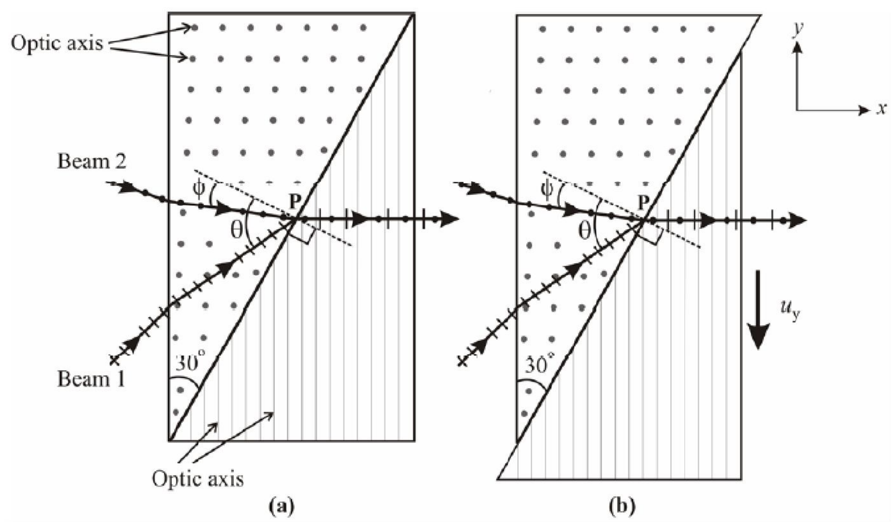
where 'c' is a constant.

(a) Expand f(x) in a Fourier series. **[Marks-12]**

(b) From the result obtained in (a). Show that **[Marks-09]**

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots\ldots\ldots\ldots$$

25. Two orthogonally polarized beams (each of wavelength $0.5\ \mu m$ and with polarization marked in the figure) are incident on a two-prism assembly and energy along x-direction, as shown. The prisms are of identical and n_0 and n_e are the refractive indices of the o-ray and e-ray, respectively. Use $\sin\phi = \frac{\sin\theta}{3}$, and $n_e = \frac{\sqrt{3}+1}{4}$



(a) Find the value of θ and n_e. **[Marks-09]**

(b) If the right hand side prism starts sliding down with the vertical component of the velocity $u_y = 1\ \mu m/s$, what would be the minimum time after which the state of polarization of the emergent beam would repeat itself? **[Marks-12]**