IIT-JAM MATHEMATICS

Test : Linear Algebra

Time : 60 Minute **Date : 23-08-2015**
M.M. : 50

Instructions:

- **Part-A** contains 10 Multiple Choice Questions (MCQ). Each question has 4 choices (a), (b), (c) and (d), for its answer, out of which **ONLY ONE** is correct. For each correct answer you will be awarded **3 marks**. For each incorrect answered **1 mark** will be deducted.
- **Part-B** contains 5 Multiple Select Questions (MSQ). Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which **ONE or MORE than ONE** is/are correct. For each correct answer you will be awarded **2 marks**, there is no negative marking in this section.
- **Part-C** contains 5 Numerical Answer Type (NAT) questions which contain **2 Marks** for each, and there is no negative marking. **Answer should be in between 0 to 9**.

PART-A

1. Let $S:\mathbb{R}^3 \to \mathbb{R}^4$ and $T:\mathbb{R}^4 \to \mathbb{R}^3$ be linear transformations such that ToS is the identity map of $\mathbb{R}^3$. Then

(a) SoT is the identity map of $\mathbb{R}^4$ (b) SoT is one-one, but not onto

(c) SoT is onto but not one-one (d) SoT is neither one-one, nor onto

2. Let V be a 3-dimensional vector space over the field $F_3 = \mathbb{Z}/3\mathbb{Z}$ of 3 elements. The number of distinct 1-dimensional subspaces of V is

(a) 13 (b) 26 (c) 9 (d) 15

3. Let $f(x)$ be the minimal polynomial of the 4 × 4 matrix

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Then rank of the 4 × 4 matrix $f(A)$ is

(a) 0 (b) 1 (c) 2 (d) 4

4. Let A and B be 3 × 3 matrices. Then AB – BA = I exists if

(a) A and B both are non-singular (b) A and B both are singular

(c) Exactly one of them is singular (d) No such A and B exists

5. The system of linear equations $x+3y=1$, $4x+py+z=0$, $2x+3z=b$ has a unique solution if and only if

$$\begin{aligned} x+3y &= 1 \\ 4x+py+z &= 0 \\ 2x+3z &= b \end{aligned}$$

(a) $a=10,\ b=-10$
(b) $a=12,\ b\in\mathbb{R}$
(c) $a\neq 10,\ b\in\mathbb{R}$
(d) $a\in\mathbb{R},\ b\neq -10$

6. Let $T:\mathbb{R}^4\to\mathbb{R}^4$ be a linear transformation satisfying $T^3+3T^2=4I$, where I is the identity transformation. Then the linear transformation $S=T^4+3T^3-4I$ is
(a) One-one but not onto
(b) Onto but not one-one
(c) Invertible
(d) Non-invertible

7. A linear transformation T rotates each vector in $\mathbb{R}^2$ clockwise through 90°. The matrix T relative to the standard ordered basis $\left\{\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix}\right\}$ is

(a) $\begin{bmatrix}0 & -1\\ -1 & 0\end{bmatrix}$
(b) $\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$
(c) $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$
(d) $\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

8. Let $T:\mathbb{R}^n\to\mathbb{R}^n$ be a linear transformation. Which of the following statements implies that T is bijective?
(a) Nullity (T) = n
(b) Rank (T) = Nullity (T) = n
(c) Rank (T) + Nullity (T) = n
(d) Rank (T) – Nullity (T) = n

9. Let $S=\{u_1,u_2,...,u_n\}$ be a linearly independent subset of a vector space V over the field Z_2. How many vectors are there in span(s)?
(a) 1
(b) 0
(c) $2n$
(d) 2^n

10. Let A be a 5 × 5 matrix with real entries such that the sum of the entries in each row of A is 1. Then the sum of all the entries in A^3 is
(a) 3
(b) 15
(c) 5
(d) 125

PART-B

11. Let V and W be finite dimensional vector spaces over $\mathbb{R}$ and let $T_1:V\to V$ and $T_2:W\to W$ be linear transformations whose minimal polynomials are given by,

$$f_1(x)=x^3+x^2+x+1 \text{ and } f_2(x)=x^4-x^2-2$$

Let $T:V\oplus W\to V\oplus W$ be the linear transformation defined by $T(V,W)=(T_1(V),T_2(W))$ for $(V,W)\in V\oplus W$ and let $f(x)$ be the minimal polynomial of T. Then
(a) $\deg f(x)=7$
(b) $\deg f(x)=5$
(c) $\text{Nullity}(T)=1$
(d) $\text{Nullity}(T)=0$

12. Let n be a positive integer and V be an $(n+1)$–dimensional vector space over $\mathbb{R}$. If $\{e_1, e_2, ..., e_{n+1}\}$ is a basis of V and $T: V \to V$ is the linear transformation satisfying

$T(e_i) = e_{i+1}$ for $i = 1, 2, ..., n$ and $T(e_{n+1}) = 0$

Then

(a) trace of T is non-zero

(b) rank of T is n

(c) nullity of T is 1

(d) $T^n = ToToTo...^{o}T$ (n times) is the zero map

13. Let A and B be $n \times n$ real matrices such that AB = BA = 0 and A + B is invertible. Which of the following are always true?

(a) $\text{rank}(A) = \text{rank}(B)$

(b) $\text{rank}(A) + \text{rank}(B) = n$

(c) $\text{nullity}(A) + \text{nullity}(B) = n$

(d) A – B is invertible

14. Let $A \in M_{10}(\mathbb{C})$, the vector space of 10 × 10 matrices with entries in $\mathbb{C}$. Let W_A be the subspace of $M_{10}(\mathbb{C})$ spanned by $\{A^n \mid n \geq 0\}$. Choose the correct statements.

(a) For any A, $\dim(W_A) \leq 10$

(b) For any A, $\dim(W_A) < 10$

(c) For some A, $10 < \dim(W_A) < 100$

(d) For some A, $\dim(W_A) = 100$

15. Which of the following are subspaces of $\mathbb{R}^2$?

(a) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 0\}$

(b) $\{(x, y) \in \mathbb{R}^2 : x^2 - y^2 = 0\}$

(c) $\{(x, y) \in \mathbb{R}^2 : xy = 0\}$

(d) $\{(x, y) \in \mathbb{R}^2 : y = 3x\}$

PART-C

16. Let $T_1, T_2 : \mathbb{R}^5 \to \mathbb{R}^3$ be linear transformations such that Rank (T_1) = 3 and Nullity (T_2) = 3. Let $T_3 : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T_3T_1 = T_2$. Then the Rank(T_3) equals ....

17. Let P and Q be two real matrices of size 4×6 and 5×4, respectively. If Rank(Q) = 4 and Rank(QP) = 2, then Rank(P) is equal to ....

18. For a fixed $a \in \mathbb{R}$, the dimension of the subspace of $P_5(\mathbb{R})$ (vector space of all polynomials of degree ≤ 5) defined by $\{f \in P_5(\mathbb{R}) : f(a) = 0\}$ is ....

19. Let $M_3(\mathbb{R})$ denote the space of all 3 × 3 real matrices. If $T : M_3(\mathbb{R}) \to M_3(\mathbb{R})$ is a linear transformation such that $T(A) = 0$ whenever $A \in M_3(\mathbb{R})$ is symmetric or skew-symmetric, then Rank(T) equals.............

20. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(1, -1) = (1, 0)$, $T(2, -1) = (0, 1)$. Then $T(-3, 2)$ equals to ....

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ANSWER KEY

PART-A

1. (d)	**2.** (a)	**3.** (a)	**4.** (d)	**5.** (c)
6. (d)	**7.** (b)	**8.** (d)	**9.** (d)	**10.** (c)

PART-B

11. (b),(d)	**12.** (b),(c)	**13.** (b),(c),(d)	**14.** (a)	**15.** (a),(d)

PART-C

16. (2)	**17.** (2)	**18.** (5)	**19.** (zero)
20. (–1, –1)			