IIT-JAM-2016

Full Length Test Series-2

TEST-VI

Duration: 3:00 Hours | **MATHEMATICS** | **Date: 20-01-2016**

Maximum Marks: 100

Read the following instructions carefully:

1. Attempt all the questions.
2. **Section-A** contains **30** Multiple Choice Questions (MCQ). Each question has 4 choices (a), (b), (c) and (d), for its answer, out of which **ONLY ONE** is correct. From **Q.1 to Q.10** carries 1 Marks and **Q.11 to Q.3**0 carries 2 Marks each.
3. **Section-B** contains **10** Multiple Select Questions(MSQ). Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which **ONE or MORE than ONE** is/are correct. For each correct answer you will be awarded **2 marks.**
4. Section-**C** contains **20** Numerical Answer Type (NAT) questions. From **Q.41 to Q.50** carries **1 Mark** each and **Q.51 to Q.60** carries **2 Marks** each. For each NAT type question, the value of answer in between 0 to 9.
5. In all sections, questions not attempted will result in zero mark. In Section–A (MCQ), wrong answer will result in negative marks. For all **1 mark** questions, **1/3 marks** will be deducted for each wrong answer. For all **2 marks** questions, **2/3 marks** will be deducted for each wrong answer. In Section–B (MSQ),there is no negative and no partial marking provisions. There is no negative marking in Section –C (NAT) as well.

Name of Student ....

College Name ....**Batch** ....

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SECTION-A : MULTIPLE CHOICE QUESTIONS (MCQ's)

Q.1 to Q.10 : Carry 1 Mark each.

1. Evaluate $\iiint_B 1\cdot dV$, where, B is the region above the xy-plane bounded by the cone $z^2=3\left(x^2+y^2\right)$ and by the sphere $x^2+y^2+z^2=1$

(a) $\frac{2\pi}{3}+\frac{\pi}{\sqrt{3}}$ (b) $\frac{\pi}{3}-\frac{\pi}{\sqrt{3}}$ (c) $-\frac{\pi}{3}+\frac{\pi}{\sqrt{3}}$ (d) none of these

2. Let $I=\int_0^\infty e^{-2x}x^6dx$ and $J=\int_0^2 x\left(8-x^3\right)^{1/3}dx$. Then $I\cdot J$ equals

(a) $\frac{45}{8}$ (b) $\frac{16}{9}\frac{\pi}{\sqrt{3}}$ (c) $\frac{10\pi}{\sqrt{3}}$ (d) $\frac{5\pi}{3\sqrt{3}}$

3. Let $x+y=u$, $y=uv$ and apply the transformation then find the $\int_0^1\int_0^{1-x} e^{\frac{y}{x+y}}dydx$

(a) $\frac{1}{2}(e-1)$ (b) $\frac{1}{2}\left(e^2-1\right)$ (c) $\frac{1}{2}(e+1)$ (d) $\frac{1}{4}\left(1-e^2\right)$

4. The vector field $\vec{v}=e^x\sin y\hat{i}+e^x\cos y\hat{j}$ is

(a) rotational (b) irrotational (c) solenoidal (d) both (b) and (c)

5. Let $H\subset\mathbb{Z}$ be a non-trivial subgroup of the group of integers $\mathbb{Z}$, we must have that

(a) the order of H is infinite (b) the order of H is less than the index of H
(c) the index of H is finite (d) the index of H is infinite

6. Which of the following is correct.

(a) $\exists$ a non-abelian group which all proper subgroup is cyclic
(b) If G is infinite cyclic group, then G has exactly two generators and G is isomorphic to the additive group of integers.
(c) every finite group of composite order possesses proper subgroups.
(d) all of the above.

7. Let A, B be $n\times n$ real matrices. Which of the following statement is correct?

(a) $Rank(A+B)=Rank(A)+Rank(B)$

(b) $Rank(A+B)\le Rank(A)+Rank(B)$

(c) $Rank(A+B)=\min\{Rank(A),Rank(B)\}$

(d) $Rank(A+B)=\max\{Rank(A),Rank(B)\}$

8. If an integral curve of the differential equation, $(y-x)\frac{dy}{dx}=1$ passes through (0, 0) and $(\alpha,1)$. Then α is equal to

(a) $2-e^{-1}$ (b) e^{-1} (c) $1-e^{-1}$ (d) $1+e$

9. Let $u_n = \frac{n^n}{n!3^n}$ then

(a) $\sum_{n=1}^{\infty} u_n$ is convergent
(b) $\sum_{n=1}^{\infty} u_n$ is divergent
(c) u_n is convergent and converges to $\frac{1}{3}$
(d) u_n is convergent and converges to 1

10. Let $S = \{(x, y) \in \mathbb{R} : x^2 + y^2 \neq 1\}$ then

(a) S is open in $\mathbb{R}^2$
(b) S is closed in $\mathbb{R}^2$
(c) S is neither open nor closed
(d) S is both open and closed

Q.11 to Q.30 : Carry 2 Marks each.

11. $\vec{A} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$, evaluate the line integral $\oint \vec{A} \cdot \overrightarrow{dr}$ from (0, 0, 0) to (1, 1, 1) along the curve C, $x = t,\ y = t^2,\ z = t^3$

(a) 5
(b) 0
(c) 1
(d) 4

12. Suppose $\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational $\vec{F} = \nabla\phi$ then $a + b - c$ equal

(a) 0
(b) 3
(c) 5
(d) 7

13. Evaluate arch length of the curve

$x = e^t \sin t,\ y = e^t \cos t$ from $t = 0$ to $t = \frac{\pi}{2}$

is

(a) $\sqrt{2}\left(e^{\frac{\pi}{2}} - 1\right)$
(b) $2\left(e^{\frac{\pi}{2}} - 1\right)$
(c) $\frac{1}{\sqrt{2}}\left(e^{\frac{\pi}{2}} - 1\right)$
(d) none of these above.

14. Evaluate the surface area of the solid generated by the revaluation of the lemniscate $r^2 = \cos 2\theta$ about the initial line

(a) $\sqrt{2}\,\pi\left(\sqrt{2} - 1\right)$
(b) $\pi\left(\sqrt{2} - 1\right)$
(c) $2\pi\left(\sqrt{2} - 1\right)$
(d) none of these

15. For $n \geq 1$, let $\left(\frac{\mathbb{Z}}{n\mathbb{Z}}\right)^*$ be the group of units of $\left(\frac{\mathbb{Z}}{n\mathbb{Z}}\right)$ which of the following groups are cyclic.

(a) $\left(\frac{\mathbb{Z}}{12\mathbb{Z}}\right)^*$
(b) $\left(\frac{\mathbb{Z}}{2^3\mathbb{Z}}\right)^*$
(c) $\left(\frac{\mathbb{Z}}{111\mathbb{Z}}\right)^*$
(d) $\left(\frac{\mathbb{Z}}{197\mathbb{Z}}\right)^*$

16. Let G be a non-abelian group of order 125 then Z(G) is

(a) 5
(b) 25
(c) 125
(d) 1

17. Let the characteristic equation of a matrix M be $\lambda^2 - \lambda - 1 = 0$ then
(a) M^{-1} does not exist
(b) M^{-1} exist but cannot be determine from the given data
(c) $M^{-1} = M + 1$ (d) $M^{-1} = M - 1$

18. The shortest distance between the curve $y^2 = x^3$ and $9x^2 + 9y^2 - 30y + 16 = 0$
(a) $\frac{\sqrt{13}}{3}$ (b) $\frac{2\sqrt{13}}{3}$ (c) $\frac{\sqrt{17}}{3}$ (d) none of these

19. $S = \{a \in \mathbb{R} : \text{'a' is a recurring decimal numbers}\}$

$T = \left\{ b \in \mathbb{R} : b = \sqrt{\frac{p}{q}} \text{ for some distinct prime p and q} \right\}$ then
(a) S is countable but T is not (b) T is countable but S is not
(c) S and T both are countable sets (d) S and T are uncountable sets

20. The solution of the differential equation $k^2 \frac{d^2y}{dx^2} = y - y_2$ under the boundary conditions
(i) $y = y_1$ at $x = 0$ (ii) $y = y_2$ at $x = \infty$
where, k, y_1 and y_2 are constant is
(a) $y = (y_1 - y_2)e^{-x/k^2} + y_2$ (b) $y = (y_2 - y_1)e^{-x/k^2} + y_1$
(c) $y = (y_1 - y_2)\sinh\left(\frac{x}{k}\right) + y_1$ (d) $y = (y_1 - y_2)e^{-x/k} + y_2$

21. Consider the differential equation

$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = 0$ with the boundary conditions of $y(0) = 0$ and $y(1) = 1$. The complete solution of the differential equation is
(a) x^2 (b) $e^x \sin\left(\frac{\pi x}{2}\right)$ (c) $\sin\left(\frac{\pi x}{2}\right)$ (d) $e^{-x} \sin\left(\frac{\pi x}{2}\right)$

22. The value of $(a + b)$ for which of the function

$$f(x) = \begin{cases} \dfrac{\sin 3x + a\sin 2x + b\sin x}{x^5} & x \neq 0 \\ c & x = 0 \end{cases}$$

is continuous at $x = 0$ the value of $a + b + c$
(a) 1 (b) 2 (c) 3 (d) 4

23. Let $F(x) = \begin{cases} \left(1 + x + \dfrac{f(x)}{x}\right)^{1/x} & x \neq 0 \\ e^3 & x = 0 \end{cases}$, $G(x) = \begin{cases} \left(1 + \dfrac{f(x)}{x}\right)^{1/x} & x \neq 0 \\ k & x = 0 \end{cases}$

where $f(x)$ is some function of x. If $F(x)$ is continuous at $x = 0$. The value of $\ln k$, so that $G(x)$ is also continuous at $x = 0$ is
(a) 0 (b) 1 (c) 2 (d) 3

24. The smallest integer n for which the permutation group S_n on n letters contains an element of order 12 is
(a) 5 (b) 9 (c) 7 (d) 11

25. Series $\sum_{n=1}^{\infty} u_n \frac{\sqrt{n+1}-\sqrt{n}}{n^{\alpha}}$ is convergent if

(a) $\alpha > \frac{1}{2}$ (b) $\alpha > \frac{2}{3}$ (c) $\alpha > \frac{3}{4}$ (d) $\alpha > \frac{4}{5}$

26. For any $a \in \mathbb{R}$, define $a_n = \frac{[a]+[2a]+.....+[na]}{n^2}$ then

(a) $\lim_{n\to\infty} a_n = \infty$ (b) $\lim_{n\to\infty} a_n = a$ (c) $\lim_{n\to\infty} a_n = \frac{a}{2}$ (d) $\lim_{n\to\infty} a_n = \frac{a}{3}$

27. Let $\{a_n\}$ be a sequence such that $0 < a_n < 1$ and $a_n(1-a_{n+1}) > \frac{1}{4}\ \forall\ n \in \mathbb{N}$ then $\{a_n\}$ is
(a) increasing and converges to 1 (b) increasing and converges to 1/2
(c) decreasing and converges to 0 (d) decreasing and converges to 1/2

28. Let $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 5 & 6 \\ 0 & 0 & 7 \end{bmatrix}$ then

(a) A is diagonalizable but not A^2 (b) A^2 is diagonalizable but not A
(c) both A and A^2 is diagonalizable (d) neither A nor A^2 is diagonalizable

29. If A is 4×4 matrix such that sum of reach row is 1 then the sum of all entries of A^6 is
(a) 4 (b) 6 (c) 64 (d) 1256

30. Consider the statement
(1) $\ell \in \mathbb{R}$ is limit point of $A \cup B$ iff l is limit point of A or limit point of B.
(2) $\ell \in \mathbb{R}$ is limit point of $A \cap B$ iff ℓ is limit point of A and limit point of B.
(a) 1 and 2 both are correct (b) 1 correct, 2 incorrect
(c) 2 correct, 1 incorrect (d) both are incorrect

SECTION-B : MULTIPLE SELECT QUESTIONS (MSQ's)

Q.31 to Q.40 : Carry 2 Marks each.

31. Evaluate $I = \int (2xyz^2)\,dx + (x^2z^2 + z\cos yz)\,dy + (2x^2yz + y\cos yz)\,dz$ from the path (1, 0, 1) to $\left(0, \frac{\pi}{2}, 1\right)$

(a) I is independent from (1, 0, 1) to $\left(0, \frac{\pi}{2}, 1\right)$

(b) I = 1

(c) I is dependent from (1, 0, 1) to $\left(0, \frac{\pi}{2}, 1\right)$

(d) I = 0

32. Which of the following is not ***TRUE***.

(a) $\int_0^4 \int_{\frac{y}{2}}^{9-y} f(x,y)\,dxdy = \int_0^2 \int_0^{2x} f(x,y)\,dydx + \int_2^5 \int_0^4 f(x,y)\,dydx + \int_5^9 \int_0^{9-x} f(x,y)\,dydx$

(b) $\int_0^1 \int_{\sqrt{2x-x^2}}^{1+\sqrt{1-x^2}} f(x,y)\,dydx = \int_0^1 dy \int_{2y-y^2}^{1+\sqrt{1-y^2}} f(x,y)\,dx$

(c) $\int_0^2 \int_{2-\sqrt{4-y^2}}^{2+\sqrt{4-y^2}} dxdy = \int_0^4 dx \int_0^{\sqrt{4-(x-1)^2}} dy$

(d) $\int_0^2 \int_1^{e^x} dxdy = \int_1^e dy \int_{\log y}^2 dx$

33. Let $u_n = \dfrac{1}{(\log n)^{\log n}}$ then

(a) $\lim\limits_{n\to\infty} u_n = \infty$
(b) $\lim\limits_{n\to\infty} u_n = 0$
(c) $\sum\limits_{n=1}^{\infty} u_n$ is convergent
(d) $\sum\limits_{n=1}^{\infty} u_n$ is divergent.

34. Let $a_n = \left(1+\dfrac{1}{n}\right)^{n+1}$; $b_n = \left(1+\dfrac{1}{n}\right)^{n}$ for $n \in \mathbb{N}$. Then

(a) $a_n < b_n \ \forall\, n$
(b) $a_n > b_n \ \forall\, n$
(c) $\{a_n\}$ is increasing seqeucne and $\{b_n\}$ is decreasing
(d) $\{a_n\}$ is decreasing and $\{b_n\}$ is increasing

35. Let A and B be subsets of $\mathbb{R}$. Define

$C = \{a+b : a \in A, b \in B\}$

Pick out the statements which is not true?
(a) C is closed if A and B are closed
(b) C is closed if A is closed and B is compact.
(c) C is compact if A is closed and B is compact
(d) none of these

36. Which of the following is not true?

(a) every countable group G has only countably many distinct subgroups
(b) every infinite abelian group has at least one element of infinite order
(c) all non-trivial proper subgroups of (R, +) are cyclic
(d) none of these

37. Let $A=\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}$ then

(a) $AA^T=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (b) $AA^T=1$

(c) A is orthogonal matrix (d) A is not orthogonal matrix

38. Let $h(x)=f(x)+[f(x)]^2+[f(x)]^3$ for every real number x. Then

(a) h is increasing whenever f is increasing (b) h is increasing whenever f is decreasing.

(c) h is decreasing whenever f is decreasing (d) nothing can be said in general.

39. A curve $y=f(x)$ passes through (1, 1) and tangent at $P(x,y)$ cuts the x-axis and y-axis at A and B respectively such that BP : AP = 3 : 1. Then

(a) equation of curve is $xy'-3y=0$

(b) normal at (1, 1) is $(x+3y)=4$

(c) curve passes through $\left(2,\frac{1}{8}\right)$

(d) equation of curve is $xy'+3y=0$

40. For every integer n, let a_n and b_n be real numbers. Let function $f:\mathbb{R}\to\mathbb{R}$ be given by

$$F(x)=\begin{cases} a_n+\sin\pi x & for\ x\in[2n,2n+1] \\ b_n+\cos\pi x & for\ x\in[2n-1,2n]\end{cases}$$

for all integer n.

(a) $a_{n-1}-b_{n-1}=0$ (b) $a_n-b_n=1$ (c) $a_n-b_{n+1}=1$ (d) $a_{n-1}-b_n=-1$

SECTION-C : NUMERICAL ANSWER TYPE (NAT's)

Q.41 to Q.50 : Carry 1 Mark each.

41. Evaluate

$$\int_0^{\pi}\int_0^{a} r^3\sin\theta\cos\theta\, dr\, d\theta$$ is ....

42. A triangular prism is formed by planes whose equations are $2y=x$, $y=0$ and $x=2$. Evaluate the volume of the prism between the planes $z=0$ and surface $z=1+xy$ ....

43. The volume of the solid generated by revolving the area of the parabola $y^2=4x$ bounded by the latus rectum about the tangent at the vertex is $k\pi$ then $[k]$ is ....

where [ ] greatest integer type equation.

44. Let $\{a_n\} = \left\{1 - \frac{2}{(n+1)(n+2)}\right\}$ then

$\lim_{n\to\infty} = a_1 a_2 ....a_n$ is ....

where [•] is greatest integer function

45. $\frac{1}{3} + \frac{1}{4.2!} + \frac{1}{5.3!} + \frac{1}{6.4!} + ...$ is ....

46. The maaximum value of the solution $y(t)$ of the differential equation $y(t) + \ddot{y}(t) = 0$ with initial conditions $\dot{y}(0) = 1$ and $y(0) = 1$ for $t \geq 0$

47. A solution of the ordinary differential equation $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 0$ is such that $y(0) = 2,\ y(1) = -\left(\frac{1-3e}{e^3}\right)$

The value of $\left|\frac{dy}{dt}(0)\right|$ is ....

48. Consider the differential equation $\frac{d^2x}{dt^2}(t) + 3\frac{dx(t)}{dt} + 2x(t) = 0$

given, $x(0) = 20,\ x(1) = \frac{10}{e},\ e = 2.718$. The value of $[x(2)]$ is ....

where [ ] greatest integer type equation.

49. The supremum of the set $\left\{\frac{n^2}{2^n}; n \in \mathbb{N}\right\}$ is ....

50. Find the number of integer less than 3600 and prime to it ....

Q.51 to Q.60 : Carry 2 Marks each.

51. $\oint_c -\frac{y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy = ?$, where $c = c_1 \cup c_2$ with $c_1 = x^2 + y^2 = 1$, $c_2 = x = \pm 2, y = \pm 2$ ....

52. $\frac{1}{\pi}\iint_s xz^2 dydz + (x^2y - z^3)dzdx + (2xy + y^2z)dxdy = ?$, where s is the surface of hemispherical region bounded by $z = \sqrt{1 - x^2 - y^2}$ and $z = 0$......................

53. Evaluate $-\frac{\sqrt{2}}{\pi}\int_c (ydx + zdy + xdz) = ?$ and c is the curve of intersection of $x^2 + y^2 + z^2 = 4$ and $x + z = 2$ ....

54. $\sum_{k=1}^{\infty} \frac{k^3 + 6k^2 + 11k + 5}{(k+3)!}$ is ....

55. Let $p(x)$ be a real polynomial of least degree which has a local maximum at $x = 1$ and a local minimum at $x = 3$. If $p(1)=6$, $p(3)=2$. Then $p'(0)$ is ....

56. Let $f:\mathbb{R}\to\mathbb{R}$ be defined by

$$f(x)=\begin{cases} x^6-1 & x\in\mathbb{Q} \\ 1-x^6 & x\notin\mathbb{Q} \end{cases}$$

The number of points at which f is continuous, is ....

57. The $\lim_{x\to 0}\frac{9}{x}\left[\frac{1}{\tan^{-1}x}-\frac{1}{x}\right]$ is equal to ....

58. Let $n\geq 1$ and let A be an n×n matrix with real enteries such that $A^k=0$ for some $k\geq 1$. Let I be the identity n×n matrix then $\det(I+A)$ is ....

59. Let $\mathbb{C}$ denote the cube $[-1,1]^3\subset\mathbb{R}^3$. How many rotations are there in $\mathbb{R}^3$ which take $\mathbb{C}$ to itself?

60. Lagrange's theorem says that, if H is a subgroup of a finite group G then the cardinality of H divides cardinality of G. The smallest number 'n' for which there exist a group of order n. For which converse of Lagrange's theorem is not tru?

IIT-JAM-MATHEMATICS **20-01-2016**

FULL LENGTH TEST -2

TEST SERIES-VI

ANSWER KEY

SECTION-A

1. (d)	2. (c)	3. (a)	4. (d)	5. (a)
6. (d)	7. (b)	8. (b)	9. (a)	10. (a)
11. (a)	12. (d)	13. (a)	14. (d)	15. (d)
16. (a)	17. (d)	18. (a)	19. (c)	20. (d)
21. (a)	22. (b)	23. (c)	24. (b)	25. (a)
26. (c)	27. (d)	28. (c)	29. (a)	30. (b)

SECTION-B

31. (a,b)	32. (b,c,d)	33. (b,c)	34. (b,d)	35. (a,c)
36. (a,b,c)	37. (a,b,c)	38. (a,c,)	39. (c,d)	40. (b,d)

SECTION-C

41. (0)	42. (1.5)	43. (3)	44. (0.333 to 0.334)	45. (0.5)
46. (1.414)	47. (3)	48. (0)	49. (1.125)	50. (960)
51. (0)	52. (0.4)	53. (4)	54. (1.666 to 1.667)	55. (6)
56. (2)	57. (3)	58. (1)	59. (24)	60. (12)