



**PHYSICS (PH)**  
**Joint Entrance Screening Test (JEST-2016)**

**Part-A: 3-Mark Questions**

1. The wavefunction of a hydrogen atom is given by the following superposition of energy eigen-functions  $\psi_{n/l/m}(\vec{r})$  ( $n, l, m$  are the usual quantum numbers);

$$\psi(\vec{r}) = \frac{\sqrt{2}}{\sqrt{7}} \psi_{100}(\vec{r}) - \frac{3}{\sqrt{14}} \psi_{210}(\vec{r}) + \frac{1}{\sqrt{14}} \psi_{322}(\vec{r})$$

The ratio of expectation value of the energy to the ground state energy and the expectation value of  $L^2$  are, respectively:

- (a)  $\frac{229}{504}$  and  $\frac{12\hbar^2}{7}$     (b)  $\frac{101}{504}$  and  $\frac{12\hbar^2}{7}$     (c)  $\frac{101}{504}$  and  $\hbar^2$     (d)  $\frac{229}{504}$  and  $\hbar^2$
2. An ideal gas with adiabatic exponent  $\gamma$  undergoes a process in which its pressure  $P$  is related to its volume  $V$  by the relation  $P = P_0 - \alpha V$ , where  $P_0$  and  $\alpha$  are positive constants. The volume starts from being very close to zero and increases monotonically to  $P_0/\alpha$ . At what value of the volume during the process does the gas have maximum entropy?
- (a)  $\frac{P_0}{\alpha(1+\gamma)}$     (b)  $\frac{\gamma P_0}{\alpha(1-\gamma)}$     (c)  $\frac{\gamma P_0}{\alpha(1+\gamma)}$     (d)  $\frac{P_0}{\alpha(1-\gamma)}$
3. The  $H_2$  molecule has a reduced mass  $M = 8.35 \times 10^{-28}$  kg and an equilibrium internuclear distance  $R = 0.742 \times 10^{-10}$  m. The rotational energy in terms of the rotational quantum number  $J$  is:
- (a)  $E_{\text{rot}}(J) = 7J(J-1)$  meV    (b)  $E_{\text{rot}}(J) = \frac{5}{2}J(J+1)$  meV
- (c)  $E_{\text{rot}}(J) = 7J(J+1)$  meV    (d)  $E_{\text{rot}}(J) = \frac{5}{2}J(J-1)$  meV
4. The Hamiltonian of a quantum particle of mass  $m$  confined to a ring of unit radius is:

$$H = \frac{\hbar^2}{2m} \left( -i \frac{\partial}{\partial \theta} - \alpha \right)^2,$$

where  $\theta$  is the angular coordinate,  $\alpha$  is a constant. The energy eigenvalues and eigenfunctions of the particle are ( $n$  is an integer):

- (a)  $\psi_n(\theta) = \frac{e^{in\theta}}{\sqrt{2\pi}}$  and  $E_n = \frac{\hbar^2}{2m}(n-\alpha)^2$     (b)  $\psi_n(\theta) = \frac{\sin(n\theta)}{\sqrt{2\pi}}$  and  $E_n = \frac{\hbar^2}{2m}(n-\alpha)^2$
- (c)  $\psi_n(\theta) = \frac{\cos(n\theta)}{\sqrt{2\pi}}$  and  $E_n = \frac{\hbar^2}{2m}(n-\alpha)^2$     (d)  $\psi_n(\theta) = \frac{e^{in\theta}}{\sqrt{2\pi}}$  and  $E_n = \frac{\hbar^2}{2m}(n+\alpha)^2$
5. Consider a quantum particle of mass  $m$  in one dimension in an infinite potential well, i.e.,  $V(x) = 0$  for  $-a/2 < x < a/2$ , and  $V(x) = \infty$  for  $|x| \geq a/2$ . A small perturbation,  $V'(x) = 2|x|/a$ , is added. The change in the ground state energy to  $O(\varepsilon)$  is:



- (a)  $\frac{\epsilon}{2\pi^2}(\pi^2 - 4)$       (b)  $\frac{\epsilon}{2\pi^2}(\pi^2 + 4)$       (c)  $\frac{\epsilon\pi^2}{2}(\pi^2 + 4)$       (d)  $\frac{\epsilon\pi^2}{2}(\pi^2 - 4)$

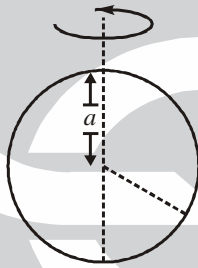
6. A spin-1 particle is in a state  $|\psi\rangle$  described by the column matrix  $(1/\sqrt{10})\{2, \sqrt{2}, 2i\}$  in the  $S_z$  basis. What is the probability that a measurement of operator  $S_z$  will yield the result  $\hbar$  for the state  $S_x|\psi\rangle$ ?

- (a)  $\frac{1}{2}$       (b)  $\frac{1}{3}$       (c)  $\frac{1}{4}$       (d)  $\frac{1}{6}$

7. The energy of a particle is given by  $E = |p| + |q|$ , where  $p$  and  $q$  are the generalized momentum and coordinate, respectively. All the states with  $E \leq E_0$  are equally probable and states with  $E > E_0$  are inaccessible. The probability density of finding the particle at coordinate  $q$ , with  $q > 0$  is:

- (a)  $\frac{(E_0 + q)}{E_0^2}$       (b)  $\frac{q}{E_0^2}$       (c)  $\frac{(E_0 - q)}{E_0^2}$       (d)  $\frac{1}{E_0}$

8. A hoop of radius  $a$  rotates with constant angular velocity  $\omega$  about the vertical axis as shown in the figure. A bead of mass  $m$  can slide on the hoop without friction. If  $g < \omega^2 a$ , at what angle  $\theta$  apart from 0 and  $\pi$  is the bead stationary (i.e.,  $\frac{d\theta}{dt} = \frac{d^2\theta}{dt^2} = 0$ )?



- (a)  $\tan \theta = \pi g / \omega^2 a$       (b)  $\sin \theta = g / \omega^2 a$       (c)  $\cos \theta = g / \omega^2 a$       (d)  $\tan \theta = g / \pi \omega^2 a$

9. A gas of  $N$  molecules of mass  $m$  is confined in a cube of volume  $V = L^3$  at temperature  $T$ . The box is in a uniform gravitational field  $-g\hat{z}$ . Assume that the potential energy of a molecule is  $U = mgz$ , where  $z \in [0, L]$  is the vertical coordinate inside the box. The pressure  $P(z)$  at height  $z$  is:

- (a)  $P(z) = \frac{N}{V} \frac{mgL}{2} \frac{\exp\left(-\frac{mg(z-L/2)}{k_B T}\right)}{\sinh\left(\frac{mgL}{2k_B T}\right)}$       (b)  $P(z) = \frac{N}{V} \frac{mgL}{2} \frac{\exp\left(-\frac{mg(z-L/2)}{k_B T}\right)}{\cosh\left(\frac{mgL}{2k_B T}\right)}$

- (c)  $P(z) = \frac{k_B T N}{V}$       (d)  $P(z) = \frac{N}{V} mgz$

10. A point charge  $q$  of mass  $m$  is released from rest at a distance  $d$  from an infinite grounded conducting plane (ignore gravity). How long does it take for the charge to hit the plane?

- (a)  $\frac{\sqrt{2\pi^3 \epsilon_0 m d^3}}{q}$       (b)  $\frac{\sqrt{2\pi^3 \epsilon_0 m d}}{q}$       (c)  $\frac{\sqrt{\pi^3 \epsilon_0 m d^3}}{q}$       (d)  $\frac{\sqrt{\pi^3 \epsilon_0 m d}}{q}$

11. The strength of magnetic field at the center of a regular hexagon with sides of length  $a$  carrying a steady current  $I$  is:



- (a)  $\frac{\mu_0 I}{\sqrt{3}\pi a}$       (b)  $\frac{\sqrt{6}\mu_0 I}{\pi a}$       (c)  $\frac{3\mu_0 I}{\pi a}$       (d)  $\frac{\sqrt{3}\mu_0 I}{\pi a}$
12. The maximum relativistic kinetic energy of  $\beta$  particles from a radioactive nucleus is equal to the rest mass energy of the particle. A magnetic field is applied perpendicular to the beam of  $\beta$  particles, which bends it to a circle of radius  $R$ . The field is given by :
- (a)  $3m_0c/eR$       (b)  $\sqrt{2}m_0c/eR$       (c)  $\sqrt{3}m_0c/eR$       (d)  $\sqrt{3}m_0c/2eR$
13. Light takes approximately 8 minutes to travel from the Sun to the Earth. Suppose in the frame of the Sun an event occurs at  $t = 0$  at the Sun and another event occurs on Earth at  $t = 1$  minute. The velocity of the inertial frame in which both these events are simultaneous is:
- (a)  $c/8$  with the velocity vector pointing from Earth to Sun  
 (b)  $c/8$  with the velocity vector pointing from Sun to Earth  
 (c) The events can never be simultaneous - no such frame exists  
 (d)  $c\sqrt{1 - (\frac{1}{8})^2}$  with velocity vector pointing from Sun to Earth
14. The central force which results in the orbit  $r = a(1 + \cos \theta)$  for a particle is proportional to :
- (a)  $r$       (b)  $r^2$       (c)  $r^{-2}$       (d) none of these
15. A spin- $\frac{1}{2}$  particle in a uniform external magnetic field has energy eigenstates  $|1\rangle$  and  $|2\rangle$ . The system is prepared in ket-state  $(|1\rangle + |2\rangle)/\sqrt{2}$  at time  $t = 0$ . It evolves to the state described by the ket  $(|1\rangle - |2\rangle)/\sqrt{2}$  in time  $T$ . The minimum energy difference between two levels is:
- (a)  $\frac{h}{6T}$       (b)  $\frac{h}{4T}$       (c)  $\frac{h}{2T}$       (d)  $\frac{h}{T}$
16. It is found that when the resistance  $R$  indicated in the figure below is changed from  $1 \text{ k}\Omega$  to  $10 \text{ k}\Omega$ , the current flowing through the resistance  $R'$  does not change. What is the value of the resistor  $R'$  ?
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- (a)  $5 \text{ k}\Omega$       (b)  $100 \text{ k}\Omega$       (c)  $10 \text{ k}\Omega$       (d)  $1 \text{ k}\Omega$
17. The sum of the infinite series  $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$  is
- (a)  $2\pi$       (b)  $\pi$       (c)  $\frac{\pi}{2}$       (d)  $\frac{\pi}{4}$
18. A spherical shell of radius  $R$  carries a constant surface charge density  $\sigma$  and is rotating about one of its diameters with an angular velocity  $\omega$ . The magnitude of the magnetic moment of the shell is:
- (a)  $4\pi\sigma\omega R^4$       (b)  $4\pi\sigma\omega R^4/3$       (c)  $4\pi\sigma\omega R^4/15$       (d)  $4\pi\sigma\omega R^4/9$
19. Given a matrix  $M = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ , which of the following represents  $\cos(\pi M/6)$ ?

(a)  $\frac{1}{2} \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$       (b)  $\frac{\sqrt{3}}{4} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$       (c)  $\frac{\sqrt{3}}{4} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$       (d)  $\frac{1}{2} \begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$

20. Consider  $N$  non-interacting electrons ( $N \sim N_A$ ) in a box of sides  $L_x, L_y, L_z$ . Assume that the dispersion relation is  $\varepsilon(k) = Ck^4$ , where  $C$  is a constant, the ratio of the ground state energy per particle to the Fermi energy is:
- (a)  $\frac{3}{7}$       (b)  $\frac{7}{3}$       (c)  $\frac{3}{5}$       (d)  $\frac{5}{7}$
21. A transistor in common base configuration has ratio of collector current to emitter current  $\beta$  and ratio of collector to base current  $\alpha$ . Which of the following is true?
- (a)  $\beta = \frac{\alpha}{(\alpha + 1)}$       (b)  $\beta = \frac{(\alpha + 1)}{\alpha}$       (c)  $\beta = \frac{\alpha}{(\alpha - 1)}$       (d)  $\beta = \frac{(\alpha - 1)}{\alpha}$
22. You receive on average 5 emails per day during a 365 days year. The number of days on average on which you do not receive any emails in that year are:
- (a) more than 5      (b) more than 2      (c) 1      (d) none of these
23. If  $Y_{xy} = \frac{1}{\sqrt{2}} (Y_{2,2} - Y_{2,-2})$ , where  $Y_{l,m}$  are spherical harmonics, then which of the following is true?
- (a)  $Y_{xy}$  is an eigenfunction of both  $L^2$  and  $L_z$       (b)  $Y_{xy}$  is an eigenfunction of  $L^2$  but not  $L_z$   
(c)  $Y_{xy}$  is an eigenfunction of  $L_z$  but not  $L^2$       (d)  $Y_{xy}$  is not an eigenfunction of either  $L^2$  or  $L_z$
24. A two dimensional box in a uniform magnetic field  $B$  contains  $N/2$  localised spin- $\frac{1}{2}$  particles with magnetic moment  $\mu$ , and  $N/2$  free spinless particles which do not interact with each other. The average energy of the system at a temperature  $T$  is:
- (a)  $3NkT - \frac{1}{2}N\mu B \sinh(\mu B/k_B T)$       (b)  $NkT - \frac{1}{2}N\mu B \tanh(\mu B/k_B T)$   
(c)  $\frac{1}{2}NkT - \frac{1}{2}N\mu B \tanh(\mu B/k_B T)$       (d)  $\frac{3}{2}NkT + \frac{1}{2}N\mu B \cosh(\mu B/k_B T)$
25. The value of the integral  $\int_0^{\infty} \frac{\ln x}{(x^2 + 1)} dx$  is:
- (a)  $\frac{\pi^2}{4}$       (b)  $\frac{\pi^2}{2}$       (c)  $\pi^2$       (d) 0

### Part-B: 1-Mark Questions

26. An ideal gas has a specific heat ratio  $C_p/C_v = 2$ . Starting at a temperature  $T_1$  the gas under goes an isothermal compression to increase its density by a factor of two. After this an adiabatic compression increases its pressure by a factor of two. The temperature of the gas at the end of the second process would be:
- (a)  $\frac{T_1}{2}$       (b)  $\sqrt{2}T_1$       (c)  $2T_1$       (d)  $\frac{T_1}{\sqrt{2}}$
27. The electric field  $\vec{E} = E_0 \sin(\omega t - kz) \hat{x} + 2E_0 \sin(\omega t - kz + \pi/2) \hat{y}$  represents:

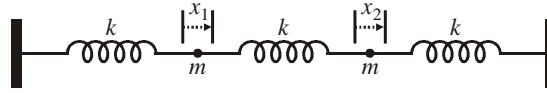


- (a) a linearly polarized wave (b) a right hand circularly polarized wave  
(c) a left hand circularly polarized wave (d) an elliptically polarized wave
28. If  $\vec{k}$  is the wavefactor of incident light ( $|\vec{k}| = 2\pi/\lambda$ ,  $\lambda$  is the wavelength of light) and  $\vec{G}$  is a reciprocal lattice vector, then the Bragg's law can be written as:  
(a)  $\vec{k} + \vec{G} = 0$  (b)  $2\vec{k} \cdot \vec{G} + G^2 = 0$  (c)  $2\vec{k} \cdot \vec{G} + k^2 = 0$  (d)  $\vec{k} \cdot \vec{G} = 0$
29. The number of different Bravais lattices possible in two dimension is:  
(a) 2 (B) 3 (c) 5 (d) 6
30. An electron confined within a thin layer of semiconductor may be treated as a free particle inside an infinitely deep one-dimensional potential well. If the difference in energies between the first and the second energy levels is  $\delta E$ , then the thickness of the layer is:  
(a)  $\sqrt{\frac{3\hbar^2\pi^2}{2m\delta E}}$  (b)  $\sqrt{\frac{2\hbar^2\pi^2}{3m\delta E}}$  (c)  $\sqrt{\frac{\hbar^2\pi^2}{2m\delta E}}$  (d)  $\sqrt{\frac{\hbar^2\pi^2}{m\delta E}}$
31. The adjoint of a differential operator  $\frac{d}{dx}$  acting on a wavefunction  $\psi(x)$  for a quantum mechanical system is:  
(a)  $\frac{d}{dx}$  (b)  $-i\hbar\frac{d}{dx}$  (c)  $-\frac{d}{dx}$  (d)  $i\hbar\frac{d}{dx}$
32. In the ground state of hydrogen atom, the most probable distance of the electron from the nucleus, in units of Bohr radius  $a_0$  is:  
(a)  $\frac{1}{2}$  (b) 1 (c) 2 (d)  $\frac{3}{2}$
33. Circular fringes are obtained with a Michelson interferometer using 600nm laser light. What minimum displacement of one mirror will make the central fringe from bright to dark ?  
(a) 600 nm (b) 300 nm (c) 150 nm (d) 120 Å
34. Given the condition  $\nabla^2\Phi = 0$ , the solution of the equation  $\nabla^2\Psi = k\vec{\nabla}\Phi \cdot \vec{\nabla}\Phi$  is given by:  
(a)  $\Psi = k\Phi^2/2$  (b)  $\Psi = k\Phi^2$  (c)  $\Psi = k\Phi \ln\Phi$  (d)  $\Psi = k\Phi \ln\Phi/2$
35. The output intensity  $I$  of radiation from a single mode of resonant cavity obeys  
$$\frac{dI}{dt} = -\frac{\omega_0}{Q}I,$$
  
where  $Q$  is the quality factor of the cavity and  $\omega_0$  is the resonant frequency. The form of the frequency spectrum of the output is:  
(a) Delta function (b) Gaussian (c) Lorentzian (d) Exponential
36. For operators  $P$  and  $Q$ , the commutator  $[P, Q^{-1}]$  is:  
(a)  $Q^{-1}[P, Q]Q^{-1}$  (b)  $-Q^{-1}[P, Q]Q^{-1}$  (c)  $Q^{-1}[P, Q]Q$  (d)  $-Q^{-1}[P, Q]Q^{-1}$
37. For a quantum mechanical harmonic oscillator with energies,  $E_n = (n + 1/2)\hbar\omega$ , where  $n = 0, 1, 2, \dots$ , the partition function is:  
(a)  $\frac{e^{\hbar\omega/k_B T}}{e^{\hbar\omega/k_B T} - 1}$  (b)  $e^{\hbar\omega/2k_B T} - 1$  (c)  $e^{\hbar\omega/2k_B T} + 1$  (d)  $\frac{e^{\hbar\omega/2k_B T}}{e^{\hbar\omega/k_B T} - 1}$

38. A semicircular piece of paper is folded to make a cone with the center of the semicircle as the apex. The half angle of the resulting cone would be:  
 (a)  $90^\circ$  (b)  $60^\circ$  (c)  $45^\circ$  (d)  $30^\circ$
39. A spin- $\frac{1}{2}$  particle is in a state  $(|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$ , where  $|\uparrow\rangle$  and  $|\downarrow\rangle$  are the eigenstates of  $S_z$  operator. The expectation value of the spin angular momentum measured along  $x$ -direction is:  
 (a)  $\hbar$  (b)  $-\hbar$  (c) 0 (d)  $\frac{\hbar}{2}$
40. The half-life of a radioactive nuclear source is 9 days. The fraction of nuclei which are left undecayed after 3 days is:  
 (a)  $\frac{7}{8}$  (b)  $\frac{1}{3}$  (c)  $\frac{5}{6}$  (d)  $\frac{1}{2^{1/3}}$
41. If the Rydberg constant of an atom of finite nuclear mass is  $\alpha R_\infty$ , where  $R_\infty$  is the Rydberg constant corresponding to an infinite nuclear mass, the ratio of the electronic to nuclear mass of the atom is:  
 (a)  $\frac{(1-\alpha)}{\alpha}$  (b)  $\frac{(\alpha-1)}{\alpha}$  (c)  $(1-\alpha)$  (d)  $\frac{1}{\alpha}$
42. A gas contains particles of type A with fraction 0.8, and particles of type B with fraction 0.2. The probability that among 3 randomly chosen particles at least one is of type A is:  
 (a) 0.8 (b) 0.25 (c) 0.33 (d) 0.992
43. A cylindrical shell of mass  $m$  has an outer radius  $b$  and an inner radius  $a$ . The moment of inertia of the shell about the axis of the cylinder is:  
 (a)  $\frac{1}{2}m(b^2 - a^2)$  (b)  $\frac{1}{2}m(b^2 + a^2)$  (c)  $m(b^2 + a^2)$  (d)  $m(b^2 - a^2)$
44. If the direction with respect to a right-handed cartesian coordinate system of the ket vector  $|z, +\rangle$  is  $(0, 0, 1)$ , then the direction of the ket vector obtained by application of rotations:  
 $\exp(-i\sigma_y\pi/2) \exp(i\sigma_x\pi/4)$ , on the ket  $|z, +\rangle$  is ( $\sigma_y, \sigma_x$  are the Pauli matrices):  
 (a)  $(0, 1, 0)$  (b)  $(1, 0, 0)$  (c)  $(1, 1, 0)/\sqrt{2}$  (d)  $(1, 1, 1)/\sqrt{3}$
45. Suppose  $yz$ -plane forms the boundary between two linear dielectric media  $I$  and  $II$  with dielectric constant  $\epsilon_I = 3$  and  $\epsilon_{II} = 4$ , respectively. If the electric field in region  $I$  at the interface is given by  $\vec{E}_I = 4\hat{x} + 3\hat{y} + 5\hat{z}$ , then the electric field  $\vec{E}_{II}$  at the interface in region  $II$  is:  
 (a)  $4\hat{x} + 3\hat{y} + 5\hat{z}$  (b)  $4\hat{x} + 0.75\hat{y} - 1.25\hat{z}$   
 (c)  $-3\hat{x} + 3\hat{y} + 5\hat{z}$  (d)  $3\hat{x} + 3\hat{y} + 5\hat{z}$
46. How much force does light from a 1.8 W laser exert when it is totally absorbed by an object?  
 (a)  $6.0 \times 10^{-9} N$  (b)  $0.6 \times 10^{-9} N$  (c)  $6.0 \times 10^{-8} N$  (d)  $4.8 \times 10^{-9} N$
47. Self inductance per unit length of a long solenoid of radius  $R$  with  $n$  turns per unit length is:  
 (a)  $\mu_0\pi R^2 n^2$  (b)  $2\mu_0\pi R^2 n$  (c)  $2\mu_0\pi R^2 n^2$  (d)  $\mu_0\pi R^2 n$
48. In Millikan's oil-drop experiment an oil drop of radius  $r$ , mass  $m$  and charge  $q = 6\pi\eta r(v_1 + v_2)/E$  is moving upwards with a terminal velocity  $v_2$  due to an applied electric field of magnitude  $E$ , where  $\eta$  is the coefficients of viscosity. The acceleration due to gravity is given by:

- (a)  $g = 6\pi\eta r v_1/m$     (b)  $g = 3\pi\eta r v_1/m$     (c)  $g = 6\pi\eta r v_2/m$     (d)  $g = 3\pi\eta r v_2/m$

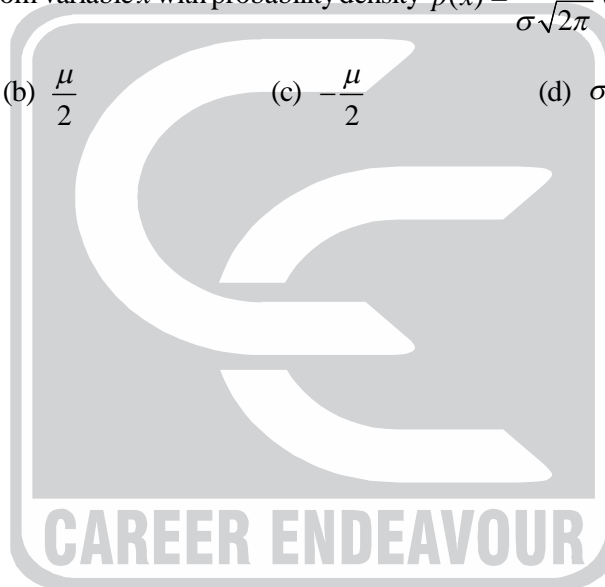
49. For the coupled system shown in the figure, the normal coordinates are  $x_1 + x_2$  and  $x_1 - x_2$ , corresponding to the normal frequencies  $\omega_0$  and  $\sqrt{3}\omega_0$ , respectively.



At  $t = 0$ , the displacements are  $x_1 = A$ ,  $x_2 = 0$ , and the velocities are  $v_1 = v_2 = 0$ . The displacement of the second particle at time  $t$  is given by:

- (a)  $x_2(t) = \frac{A}{2} \left( \cos(\omega_0 t) + \cos(\sqrt{3}\omega_0 t) \right)$     (b)  $x_2(t) = \frac{A}{2} \left( \cos(\omega_0 t) - \cos(\sqrt{3}\omega_0 t) \right)$   
 (c)  $x_2(t) = \frac{A}{2} \left( \sin(\omega_0 t) - \sin(\sqrt{3}\omega_0 t) \right)$     (d)  $x_2(t) = \frac{A}{2} \left( \sin(\omega_0 t) - \frac{1}{\sqrt{3}} \sin(\sqrt{3}\omega_0 t) \right)$
50. The mean value of random variable  $x$  with probability density  $p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left[ -(x^2 + \mu x)/(2\sigma^2) \right]$ , is:

- (a) 0    (b)  $\frac{\mu}{2}$     (c)  $-\frac{\mu}{2}$     (d)  $\sigma$



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