Target IIT-JAM-2017

Test Series-1

Booklet Code: A

Sequences and Series of Real Numbers + Function of One or two variables

Duration: 2:00 Hours | **MATHEMATICS-MA** | **Date: 29-12-2016**

Maximum Marks: 100

Read the following instructions carefully:

1 Attempt all the questions.

2. **Section-A** contains **30** Multiple Choice Questions (MCQ). Each question has 4 choices (a), (b), (c) and (d), for its answer, out of which **ONLY ONE** is correct. From **Q.1 to Q.10** carries 1 Marks and **Q.11 to Q.3**0 carries 2 Marks each.

3. **Section-B** contains **10** Multiple Select Questions(MSQ). Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which **ONE or MORE than ONE** is/are correct. For each correct answer you will be awarded **2 marks.**

4. Section**-C** contains **20** Numerical Answer Type (NAT) questions. From **Q.41 to Q.50** carries **1 Mark** each and **Q.51 to Q.60** carries **2 Marks** each. For each NAT type question, the value of answer in between 0 to 9.

5. In all sections, questions not attempted will result in zero mark. In Section–A (MCQ), wrong answer will result in negative marks. For all **1 mark** questions, **1/3 marks** will be deducted for each wrong answer. For all **2 marks** questions, **2/3 marks** will be deducted for each wrong answer. In Section–B (MSQ),there is no negative and no partial marking provision. There is no negative marking in Section –C (NAT) as well.

Regn. No. | | | | | | | | | |

Name of Student ....**Batch**...........................

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SECTION-A : MULTIPLE CHOICE QUESTIONS (MCQ's)

Q.1 to Q.10 : Carry 1 Mark each.

1. Let $\{x_n\}$ be a real sequence. If sequence of even terms of $\{x_n\}$ converges to 1 and sequence of odd terms converges to -1 then the sequence $\{x_n\}$ will be
   (a) converge to 0 (b) converge to 1 (c) converge to -1 (d) none of these

2. The limit superior and the limit inferior of the following sequence $\left\langle (-1)^n \left(1+\frac{1}{n}\right)\right\rangle$ are
   (a) 1, –1 (b) 2, –1 (c) 2, 1 (d) 1, 1

3. Let $\langle x_n \rangle$ be a convergent sequence and $\langle y_n \rangle$ be a monotonic sequence. Then $\langle x_n \cdot y_n \rangle$
   (a) always converges
   (b) converges if $\langle x_n \rangle$ is monotonic
   (c) converges if $\langle x_n \cdot y_n \rangle$ monotonic
   (d) converges if $\langle y_n \rangle$ is bounded

4. The set $\left\{\frac{1}{n}\sin\frac{1}{n} \mid n \in \mathbb{N}\right\}$ has
   (a) one limit point and it is 0
   (b) one limit point and it is 1
   (c) one limit point and it is –1
   (d) three limit points and these are –1, 0 and 1

5. $\sum \frac{n^n}{(n+a)^n} : a > 0$
   (a) converges for all $a > 1$
   (b) converges for $a > 0$
   (c) converges for $a \geq 0$
   (d) divergent for all a

6. Maximum value of xyz under the constraints $x^2 + z^2 = 1$ and $y - x = 0$
   (a) $\frac{2}{\sqrt{3}}$ (b) $\frac{2}{3\sqrt{3}}$ (c) $\frac{4}{\sqrt{3}}$ (d) $\frac{2}{4\sqrt{3}}$

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7. $f(x,y) = x^2 - 2xy + y^2 + x^3 - y^3 + x^5$ has at point (0, 0)
   (a) maxima
   (b) minima
   (c) neither maxima nor minima
   (d) none

8. $f(x,y) = \begin{cases} \frac{x^2+y^2}{x-y} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ have
   (a) continuity at (0, 0) and posses partial derivative at (0, 0)
   (b) discontinuous but posses partial derivative at (0, 0)
   (c) does not posses partial derivative at (0, 0)
   (d) none of these

9. $f(x,y)=\sqrt{x^2+y^2}$ is

(a) continuous and differentiable at (0, 0)
(b) discontinuous but differentiable at (0, 0)
(c) continuous but non-differentiable at (0, 0)
(d) discontinuous at (0, 0)

10. Let $f(x,y)=\begin{cases}\frac{x}{|x|}\sqrt{x^2+y^2} & \text{if } x\neq 0\\ 0 & \text{if } x=0\end{cases}$ then $f_x(0,0)+f_y(0,0)$ equals

(a) 1
(b) –1
(c) –2
(d) 0

Q.11 to Q.30 : Carry 2 Marks each.

11. Consider the sequence $\{\ell n\}$ $n\in\mathbb{N}$ with $\ell n=\frac{1}{n+1}+\cdots+\frac{1}{2n}$. This sequence

(a) is increasing and bounded
(b) increasing to ∞
(c) decreases to $-\infty$
(d) decreases to a positive number

12. The largest term in sequence $x_n=\frac{1000^n}{n!}$, $n=1,2,3,\ldots.$

(a) is x_{999}
(b) is x_{1001}
(c) is x_1
(d) does not exist

13. Setect the CORRECT statement:

(a) Finite set may have infinite number of limit points
(b) The sequence $1+r+r^2+\cdots r^n$, where $-1\le r<1$ and $n\in\mathbb{N}$ is bounded above but not bounded below
(c) If $u_n>0\ \forall\ n$ then $\sum u_n$ and $\sum\frac{1}{u_n}$ converge or diverge together
(d) Set of real number except the integers which are multiples of 2 is uncountable set

14. If $\sum a_n$ be a convergent series and $a_n>0\ \forall\ n\in\mathbb{N}$ then the INCORRECT statement.

(a) $\sum\sqrt{a_n\,a_{n+1}}$ is convergent
(b) $\sum a_n^2$ is convergent
(c) $\sum\frac{\sqrt{a_n}}{n}$ is convergent
(d) None of these

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15. Pick out the series which are absolutely convergent.

(a) $\sum_{n=1}^{\infty}(-1)^n\frac{\cos n\alpha}{n^2}$ where $\alpha\in\mathbb{R}$ is a fixed real number

(b) $\sum_{n=1}^{\infty}(-1)^n\frac{n\log n}{e^n}$

(c) $\sum_{n=1}^{\infty}(-1)^n\frac{n}{n+2}$

(d) None of these

16. If $b_n = \begin{cases} \frac{1}{\sqrt{n}} & \text{if } n \text{ is odd} \\ \frac{1}{n} & \text{if } n \text{ is even} \end{cases}$. Then,

(a) Both $\langle b_n \rangle$ and $\sum b_n$ are convergent
(b) Both $\langle b_n \rangle$ and $\sum b_n$ are divergent
(c) $\langle b_n \rangle$ is convergent but $\sum b_n$ is not
(d) None of these

17. $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \frac{1}{3n+k}$ is

(a) $\log \frac{4}{3}$ (b) $\log \frac{3}{4}$ (c) $\log \frac{3}{2}$ (d) $\log \frac{5}{4}$

18. If $S = \left\{ \frac{1}{2m} + \frac{1}{2n} \mid m, n \in \mathbb{N} \right\}$, then the derived set S' and S'' has the property

(a) S' and S'' both are finite
(b) S' is finite and S'' is empty
(c) S' is infinite and S'' is singleton
(d) S' and S'' both are infinite

19. Let a_1, a_2 be positive real numbers and let $a_{n+1} = \frac{1}{2}(a_n + a_{n-1})$ for $n \geq 2$, then the sequence $\{a_{2n}\}$ and $\{a_{2n-1}\}$

(a) are not bounded
(b) both diverge to ∞
(c) both converge to the same limit
(d) both converge but different limits

20. Let $x_n = 2^{2n}\left(1 - \cos\left(\frac{1}{2^n}\right)\right) \forall n \in \mathbb{N}$. Then the sequence $\{x_n\}$

(a) does not converge (b) converges to 0 (c) converge to 1/2 (d) converge to 1/4

21. The function f defined on R by $f(x) = 3^x + 4^x - 5^x$ has

(a) exactly one root
(b) exactly two roots
(c) exactly three roots
(d) infinitely many roots

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22. The number of local minimum of $f(x) = \left|x - \frac{1}{2}\right| + \left|x + \frac{1}{2}\right|, x \in R$

(a) 0 (b) 2 (c) 1 (d) infinitely many

23. If $f(x) = ax^3 + bx^2 + x + 1$ has local maximum value 3 at $x = -2$ then $4a + 2b$ is equal to

(a) 6 (b) 8 (c) 7 (d) none

24. If $f(x, y) = \begin{cases} \frac{2x^3 + 3y^3}{x^2 + y^2}, & (x, y) \neq (0,0) \\ 0, & (x, y) = (0,0) \end{cases}$ then $\{f_x(0,0) + f_y(0,0)\}$ is equal to

(a) –5 (b) 4 (c) –4 (d) 5

25. For $(x, y) \in \mathbb{R}^2$, let $f(x,y) = \begin{cases} \dfrac{2xy}{x^2+y^2}; & \text{if}(x,y) \neq (0,0) \\ 0; & \text{if}(x,y) = (0,0) \end{cases}$

Then,

(a) f_x and f_y exist at (0, 0) and f is continuous at (0, 0)

(b) f_x and f_y exist at (0, 0) and f is discontinuous at (0, 0)

(c) f_x and f_y do not exist at (0, 0) and f is continuous at (0, 0)

(d) f_x and f_y do not exist at (0, 0) and f is discontinuous at (0, 0)

26. The continuous function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = (x^2+1)^{57}$ is

(a) Onto but not one-one
(b) One-one but not onto
(c) Both one-one and onto
(d) Neither one-one nor onto

27. If x, y and z are positive real numbers, then the minimum value of $x^2 + 8y^2 + 27z^2$, where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ is

(a) 108
(b) 216
(c) 405
(d) 1048

28. Let $f(x,y) = x^5 \dfrac{e^{y/x}(x+y)}{(x^2+y^2)^{3/2}}$. Then $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y}$ equals

(a) $2f$
(b) $3f$
(c) $5f$
(d) $7f$

29. Maximum value of $f(x,y,z) = xyz$ along all points lying on the intersection of planes $x + y + z = 40$ and $z = x + y$ is

(a) 4000
(b) 2000
(c) 3000
(d) 1000

30. If $z = e^{xy^2}, x = t\cos t, y = t\sin t$ then $\dfrac{dz}{dt}$ at $t = \dfrac{\pi}{2}$ is

(a) $\dfrac{\pi^3}{8}$
(b) $\dfrac{\pi^3}{4}$
(c) $\dfrac{\pi^3}{2}$
(d) $\dfrac{-\pi^3}{8}$

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SECTION-B : MULTIPLE SELECT QUESTIONS (MSQ's)

Q.31 to Q.40 : Carry 2 Marks each.

31. Let $\{a_0, a_1, a_2, ...\}$ be a sequence of real numbers for any $k \geq 1$. Let $S_n = \sum_{k=0}^{n} a_{2k}$ which of the following statement(s) are CORRECT ?

(a) If $\lim_{n\to\infty} S_n$ exists, then $\sum_{m=0}^{\infty} a_m$ exists
(b) If $\lim_{n\to\infty} S_n$ exists, then $\sum_{m=0}^{\infty} a_m$ need not exists
(c) If $\sum_{m=0}^{\infty} a_m$ exists, then $\lim_{n\to\infty} S_n$ exists
(d) If $\sum_{m=0}^{\infty} a_m$ exists, then $\lim_{n\to\infty} S_n$ need not exists

32. Consider the following statement(s) and choose correct ?
(a) Every infinite bounded set must have limit point
(b) Any finite set cannot have limit point
(c) Any infinite unbounded set can't have limit point
(d) None of these

33. Let $\langle a_n \rangle$ and $\langle b_n \rangle$ are two sequences then choose the INCORRECT ?
(a) If $\langle a_n + b_n \rangle$ and $\langle a_n \cdot b_n \rangle$ are both convergent then $\langle a_n \rangle$ and $\langle b_n \rangle$ are convergent
(b) If $\langle a_n - b_n \rangle$ and $\langle a_n / b_n \rangle$ are both convergent then $\langle a_n \rangle$ and $\langle b_n \rangle$ are convergent
(c) If $\langle a_n + b_n \rangle$ and $\langle a_n - b_n \rangle$ are both convergent then $\langle a_n \rangle$ and $\langle b_n \rangle$ are convergent
(d) If $\langle a_n + b_n \rangle$ and $\langle a_n - b_n \rangle$ are both convergent then $\langle a_n \cdot b_n \rangle$ convergent

34. Choose the correct answer where $a_n \geq 0$
(a) If $\sum a_n$ is convergent then $\sum a_n^2$ is also convergent
(b) If $\sum a_n$ is convergent then $\sum \sqrt{a_n}$ is also convergent
(c) If $\sum a_n$ is convergent then $\sum \frac{a_n}{1+a_n}$ is also convergent
(d) If $\sum a_n$ is convergent then $\sum \frac{a_n}{1-a_n}$ (provided $a_n \neq 1$) is also convergent

35. Which of the following are null sequences
(a) $\left\{\frac{n}{\alpha^n}\right\} : \alpha > 1$ (b) $\left\{\frac{\log n}{n}\right\}$ (c) $\left\{\alpha^n : |\alpha| < 1\right\}$ (d) $\left(1+\frac{3}{n}\right)^n \mid n \in \mathbb{N}$

36. Let $f(x) = 2x^3 - x^2 + 2x - 5$ then for $f(x) = 0$
(a) at least one root is positive
(b) at least one root is negative
(c) there is a root between $x = 1$ and $x = 2$
(d) none

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37. Let $u = (x+y+z)^3 - 3(x+y+z) - 24xyz + a^3$ has
(a) minimum at (1, 1, 1) (b) maximum at (–1, –1, –1)
(c) maximum at (1, 1, 1) (d) minimum at (–1, –1, –1)

38. $$f(x,y) = \begin{cases} \frac{xy}{|x|} & \text{if } x \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) $f(x, y)$ is continuous at (0, 0) (b) $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exists at (0, 0)
(c) $f(x, y)$ is discontinuous at (0, 0) (d) none

39. Let $f(x) = \dfrac{1}{8 - 3\sin x}$

(a) domain of $f(x)$ is R

(b) Range of $f(x)$ is $\left[\dfrac{1}{10}, \dfrac{1}{5}\right]$

(c) Range of $f(x)$ is $\left[\dfrac{1}{11}, \dfrac{1}{5}\right]$

(d) domain is $R - \left\{\dfrac{8}{3}\right\}$

40. $\lim\limits_{n\to\infty} \dfrac{2^{n+1} + 5^{n+1}}{2^n + 5^n}$

(a) limit does not exist

(b) limit exists

(c) limit equals 5

(d) limit equals 2

SECTION-C : NUMERICAL ANSWER TYPE (NAT's)

Q.41 to Q.50 : Carry 1 Mark each.

41. If p is a real number, then the series $\dfrac{1}{1^p} + \dfrac{1}{3^p} + \dfrac{1}{5^p} + \dfrac{1}{7^p} + \cdots$ is convergent for $p >$ ....

42. Let $p(x)$ be a polynomial in the real variable x of degree 5. Then $\lim\limits_{n\to\infty} \dfrac{p(n)}{2^n}$ is ....

43. Let $A = \left\{ x \mid x = \dfrac{4n+3}{n}; n \in \mathbb{N} \right\}$, then infimum of A is ....

44. $S = \left\{ (-1)^n \left(\dfrac{1}{4} - \dfrac{4}{n} \right) \middle| n \in \mathbb{N} \right\}$, then sup S is ....

45. Value of $\sum\limits_{n=1}^{\infty} \dfrac{n}{e^n}$ is ....

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46. Greatest value of function $f(x) = x^3 - 12x,\ x \in [-1, 3]$ ....

47. $f(x) = 3(x-2)^{2/3} - (x-2),\ 0 \le x \le 20$, a_1 and a_2 are points of global minima and maxima, respectively of $f(x)$ in interval [0, 20]. Then $f(a_1) + f(a_2)$ is equal to ....

48. $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{\sqrt{x^2 + y^2}}$ is equal to ....

49. Let $f(x,y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2} & (x,y) \ne (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ then $f_{xy}(0,0) + f_{yx}(0,0)$ is equal to ....

50. $\left(xy+\frac{9}{x}+\frac{3}{y}\right)$ has minimum value ....

Q.51 to Q.60 : Carry 2 Marks each.

51. Define a sequence S_n by $S_n=\sum_{k=1}^{n}\frac{1}{\sqrt{n^2+k}}$, then the limit of S_n as n tends to infinity is ....

52. Suppose $a>0$, consider the sequence $a_n=n\left(\sqrt[n]{ea}-\sqrt[n]{a}\right), n\geq 1$ then $\lim_{n\to\infty} a_n$ is ....

53. $\lim_{n\to\infty}\left[\left(\frac{2}{1}\right)\left(\frac{3}{2}\right)^2\left(\frac{4}{3}\right)^3\cdots\left(\frac{n+1}{n}\right)^n\right]^{1/n}$ is ....

54. $\lim_{n\to\infty}\frac{1}{n^4}\sum_{J=0}^{2n-1}J^3$ equals ....

55. Let $\{a_n\}$ be a sequence of positive real numbers such that $\lim_{n\to\infty}\frac{a_n}{a_{n+1}}>1$ then $\lim_{n\to\infty} e^{a_n^2+a_{n+1}}$ is ....

56. Sum of absolute minimum and absolute maximum values of $f(x,y)=(x+y)^2-(x+y)+1$ on a unit square $\{(x,y):0<x<1, 0<y<1\}$ ....

57. Let $f:(0,\infty)\to(0,\infty)$ be a continuously differentiable function and let $z=\frac{x}{f(x^3+y^3)}$, if f is homogeneous of degree 1, then $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=kz$ then find k ....

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58. Let $f(x,y)=\begin{cases}\frac{xy(x^2-y^2)}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$ then value of $\{f_{xy}(0,0)+f_{yx}(0,0)\}$ is equal to ....

59. Let $f(x,y)=\begin{cases}\frac{xy}{(x^2+y^2)\times\frac{5}{2}}[1-\cos(x^2+y^2)]: & (x,y)\neq(0,0)\\ K: & (x,y)=(0,0)\end{cases}$

Then, the value of K for which $f(x,y)$ is continuous at (0, 0) is ....

60. $\lim_{n\to\infty}\frac{4^{n+1}+5^{n+1}}{4^n+5^n}$ is equal to ....

****** **END** ******

Space for rough work

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TEST SERIES - 1

Sequences and Series of Real Numbers + Function of One or two variable

ANSWER KEY

SECTION-A

1. (d)	2. (a)	3. (d)	4. (a)	5. (d)
6. (b)	7. (c)	8. (a)	9. (c)	10. (c)
11. (a)	12. (a)	13. (d)	14. (d)	15. (a)
16. (c)	17. (a)	18. (c)	19. (c)	20. (c)
21. (a)	22. (d)	23. (b)	24. (d)	25. (b)
26. (d)	27. (b)	28. (b)	29. (b)	30. (d)

SECTION-B

31. (b, d)	32. (a, b)	33. (a, b)	34. (a, b, c, d)	35. (a, b, c)
36. (a, b)	37. (a, b)	38. (a, b)	39. (a, c)	40. (b, c)

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SECTION-C

41. (1)	42. (0)	43. (4)	44. (3.75)	45. (0.921)
46. (11)	47. (11.36)	48. (0)	49. (0)	50. (9)
51. (1)	52. (1)	53. (2.71)	54. (4)	55. (1)
56. (3.75)	57. ($k = -2$)	58. (0)	59. (0)	60. (5)

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