Target IIT-JAM-2017

Test Series-2

Booklet Code: B

Integral Calculus + Group Theory + Vector Calculus

Duration: 2:00 Hours **MATHEMATICS-MA** **Date: 08-01-2017**

Maximum Marks: 100

Read the following instructions carefully:

1 Attempt all the questions.

2. **Section-A** contains **30** Multiple Choice Questions (MCQ). Each question has 4 choices (a), (b), (c) and (d), for its answer, out of which **ONLY ONE** is correct. From **Q.1 to Q.10** carries 1 Marks and **Q.11 to Q.3**0 carries 2 Marks each.

3. **Section-B** contains **10** Multiple Select Questions(MSQ). Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which **ONE or MORE than ONE** is/are correct. For each correct answer you will be awarded **2 marks.**

4. Section**-C** contains **20** Numerical Answer Type (NAT) questions. From **Q.41 to Q.50** carries **1 Mark** each and **Q.51 to Q.60** carries **2 Marks** each. For each NAT type question, the value of answer in between 0 to 9.

5. In all sections, questions not attempted will result in zero mark. In Section–A (MCQ), wrong answer will result in negative marks. For all **1 mark** questions, **1/3 marks** will be deducted for each wrong answer. For all **2 marks** questions, **2/3 marks** will be deducted for each wrong answer. In Section–B (MSQ),there is no negative and no partial marking provision. There is no negative marking in Section –C (NAT) as well.

Regn. No. | | | | | | | | | | |

Name of Student ....**Batch**............................

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SECTION-A : MULTIPLE CHOICE QUESTIONS (MCQ's)

Q.1 to Q.10 : Carry 1 Mark each.

1. Determine the volume generated when the area above the x-axis bounded by the curve $x^2+y^2=9$ and the co-ordinates $x=3$ and $x=-3$ is rotated.

(a) 18π (b) 36π (c) 54π (d) 9π

2. Find the area of the plane bounded by the curve $y=(x-1)^2$ and $y=4-(x-3)^2$

(a) $\frac{2}{3}$ (b) $\frac{5}{3}$ (c) $\frac{1}{3}$ (d) $\frac{8}{3}$

3. Find the area of the surface $z=\sqrt{x^2+y^2}$ over the region bounded by $x^2+y^2=1$

(a) π (b) $\frac{\pi}{\sqrt{2}}$ (c) $\sqrt{2}\,\pi$ (d) $2\sqrt{2}\,\pi$

4. Find the arc length of the helix $r(t)=\cos t\hat{i}+\sin t\,\hat{j}+2t\hat{k}$ between the points $t=0$ to $t=2\pi$

(a) $\pi\sqrt{5}$ (b) $\sqrt{2}\,\pi$ (c) $2\sqrt{2}\,\pi$ (d) $2\sqrt{5}\,\pi$

5. Find the line integral of $F=-yz^2\,\hat{i}+xz^2\,\hat{j}+yz\,\hat{k}$ along the helix $r(t)=\cos t\,\hat{i}+\sin t\,\hat{j}+t\,\hat{k}$ from $t=0$ to $t=2\pi$

(a) 2π (b) -2π (c) $\frac{8}{3}\pi^3-2\pi$ (d) $\frac{4}{3}\pi^3-2\pi$

6. Find the flux of $\vec{F}=(2x+y)\hat{i}+(y-x)\hat{j}$ across the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$

(a) 18π (b) 9π (c) 6π (d) 27π

7. If $E=\{1, 2, 3, 4\}$ and $F=\{1, 2\}$, then the numbers of onto function from E to F is

(a) 14 (b) 16 (c) 12 (d) 8

8. The domain of definition of the function $y(x)$ as given by the equation $2^x+2^y=2$ is

(a) $0<x\le 1$ (b) $0\le x\le 1$ (c) $-\infty<x\le 0$ (d) $-\infty<x<1$

9. If x and y are two sets then $x\cap(x\cup y)^c$ equals

(a) x (b) y (c) ϕ (d) none of these

10. Consider the group $\mathbb{Z}_p\oplus\mathbb{Z}_p$ under addition. The numbers of cyclic subgroups of order p is

(a) 1 (b) $p-1$ (c) $p+1$ (d) p^2-1

Q.11 to Q.30 : Carry 2 Marks each.

11. Let $x = u^2 - v^2$, $y = 2v$ then $I = \iint_R \left(x^2 + y^2\right) dxdy$ is equivalent to

(a) $4\iint_R \left(u^2 + v^2\right)^3 du\, dv$ (b) $2\iint_R \left(u^2 + v^2\right)^3 du\, dv$

(c) $\iint_R \left(u^2 + v^2\right) du\, dv$ (d) $\frac{1}{2}\iint_R \left(u^2 + v^2\right)^3 du\, dv$

12. Let R be the region in $\mathbb{R}^2$ determined by the inequalities $x^2 + y^2 \le 4$ and $y^2 \le x^2$, evaluate the following integral $\iint_R \sin\left(x^2 + y^2\right) dxdy$

(a) $\frac{\pi}{2}(1 - \cos 4)$ (b) $\frac{\pi}{2}(1 + \cos 4)$ (c) $\frac{\pi}{4}(1 - \cos 4)$ (d) $\frac{\pi}{4}(1 + \cos 4)$

13. Let R be the region $\mathbb{R}^3$ determined by the inequalities $r \le 1$ and $0 \le z \le 4 - r^2$ where $r = \sqrt{x^2 + y^2}$, evaluate $\iiint_R rdV$

(a) $\frac{17}{15}\pi$ (b) $\frac{34}{15}\pi$ (c) $\frac{51}{15}\pi$ (d) none

14. If area bounded by the curve $y = x^3$ and $y = x, x \ge 0$ is revolved around x-axis then volume generated is

(a) $\frac{\pi}{21}$ (b) $\frac{2\pi}{21}$ (c) $\frac{3\pi}{21}$ (d) $\frac{4\pi}{21}$

15. Determine the arc length of the curve

$$x = \frac{1}{3}\sqrt{y}\,(y - 3),\ 1 \le y \le 9$$

(a) $\frac{16}{3}$ (b) $\frac{32}{3}$ (c) $\frac{8}{3}$ (d) $\frac{4}{3}$

16. Evaluate $\iint_R (x + y)\, dA$, where R is the trapezoidal region with vertices given by (0, 0), (5, 0), $\left(\frac{5}{2}, \frac{5}{2}\right)$, $\left(\frac{5}{2}, -\frac{5}{2}\right)$

(a) $\frac{125}{48}$ (b) $\frac{125}{4}$ (c) $\frac{125}{24}$ (d) none

Hint : $x = 2u + 3v$; $y = 2u - 3v$

17. Evaluate $\iint_S x^3 dy\, dz + y^3 dx\, dz + z^3 dx\, dy$ where S is the boundary of the volume V occupying the region between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ and above the plane $z = 0$

(a) $\frac{62}{5}\pi$ (b) $\frac{186}{5}\pi$ (c) $\frac{31}{5}\pi$ (d) $\frac{93}{5}\pi$

18. Let S be the surface of the paraboloid of revolution $z = 1 - x^2 - y^2$ with the domain of definition $x^2 + y^2 \le 1$ and Γ be the boundary of the paraboloid. Given $\vec{F} = x^3\hat{i} + (x + y - z)\hat{j} + yz\hat{k}$, then $\iint_S \text{curl}\, F \cdot ds$

(a) 0 (b) 2π (c) π (d) $\frac{\pi}{2}$

19. Evaluate $I = \int_C (x^2 + y)dx + (x - y^2)dy$ from $A = (0, 2)$ to $B = (3, 5)$ along the curve $y = 2 + x$

(a) 15 (b) –15 (c) 30 (d) 0

20. Find the directional derivative of $f(x, y, z) = x^2 + 3y^2 + 2z^2$ in the direction of vector $2\hat{i} - \hat{j} - 2\hat{k}$ at $(1, -3, 2)$

(a) 6 (b) 2 (c) –6 (d) –2

21. Evaluate $I = \oint_c (2x - y)dx + (2y + x)dy$ around the boundary c of the ellipse $x^2 + 9y^2 = 16$

(a) $\frac{16\pi}{3}$ (b) $\frac{8\pi}{3}$ (c) $\frac{32\pi}{3}$ (d) $\frac{4\pi}{3}$

22. A surface S consist of that part of the cylinder $x^2 + y^2 = 9$ between $z = 0$ and $z = 4$ for $y \ge 0$ and the two semi circle of radius 3 in the plane $z = 0$ and $z = 4$. If $F = z\hat{i} + xy\hat{j} + xz\hat{k}$ evaluate $\iint_S \text{curl}\, F \cdot dS$

(a) 24 (b) 0 (c) 12 (d) none

23. If n is a positive integer such that the sum of all positive integers a satisfying $1 \le a \le n$ and gcd $(a, n) = 1$ is equal to 240, then the number of summands, namely $\phi(n)$ is

(a) 120 (b) 124 (c) 240 (d) 480

24. The number of multiples of 10^{44} that divide 10^{55} is

(a) 11 (b) 12 (c) 121 (d) 144

25. Which of the following is not a subgroup of $(\mathbb{C}, +)$

(a) $(\mathbb{R}, +)$

(b) $(G, +)$, where $G = \{\pi r \mid r \in Q\}$

(c) $(G, +)$, where $G = \{i r \mid r \in \mathbb{R}\}$

(d) $(G, +)$, where $G = \{\pi^n \mid n \in \mathbb{Z}\}$

26. Let $\mathbb{Q}$ be the set of rational numbers and let $G = \mathbb{Q}^c \cup \{0\}$. Then under the usual addition of real numbers, G is

(a) A group, since $\mathbb{R}$ and $\mathbb{Q}$ are groups under addition

(b) A group, since additive identity is in G

(c) Not a group, since addition on G is not a binary operation

(d) Not a group, since not all elements in G have an inverse

27. In the group $\{1, 2, ..., 16\}$ under the operation of multiplication modulo 17, the order of the element 3 is

(a) 4 (b) 8 (c) 12 (d) 16

28. Let $a = (123)\,(45) \in S_5$ and $b = (23)\,(14) \in S_5$, then aba^{-1} is equal to

(a) (13) (25) (b) (135) (24) (c) (15) (23) (d) (123) (45)

29. Evaluate $\int_0^{\pi/2}\int_0^{\cos x} x^2 dydx$

(a) $\frac{\pi^2}{4}$ (b) -2 (c) $\frac{\pi^2}{4}-2$ (d) $\frac{\pi^2}{4}+2$

30. Let C be the rectangle in $\mathbb{R}^3$ with vertices (0, 0, 0), (1, 0, 0), (1, 1, 1) and (0, 1, 1) oriented in the given order then absolute value of $\oint_C \sin x^2 dx + xy^2 dy + xz^2 dz$

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

SECTION-B : MULTIPLE SELECT QUESTIONS (MSQ's)

Q.31 to Q.40 : Carry 2 Marks each.

31. Which of the following is true.

(a) $\vec{F} = y^2 z\,\hat{i} + 2xyz\,\hat{j} + \left(2z + xy^2\right)\hat{k}$ is conservative field.

(b) $\vec{F} = 2xyz\,\hat{i} + x^2 z\,\hat{j} + x^2 y\,\hat{k}$ is conservative field.

(c) $\vec{F} = 2xy\,\hat{i} + \left(x^2 - y\right)\hat{j}$ is not conservative field.

(d) $\vec{F} = yz\,\hat{i} + xz\,\hat{j} + xy\,\hat{k}$ is not conservative field

32. Which of the following is true?

(a) Let $\phi = f(r)$ then $\nabla^2\phi = f''(r) + \frac{1}{r} f'(r)$

(b) Let $\phi = f(r)$ then $\nabla^2\phi = f''(r) + \frac{2}{r} f'(r)$

(c) $\vec{F} = yz^2\,\hat{i} + xy\,\hat{j} + yz\,\hat{k}$ then curl $\vec{F}$ is solenoidal

(d) $\vec{F} = yz^2\,\hat{i} + xy\,\hat{j} + yz\,\hat{k}$ then curl $\vec{F}$ is not solenoidal

33. Which of the following is not true

(a) If $\vec{a}$ is constant vector, $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$ then, $\nabla(\vec{a}\cdot\vec{r}) = 0$

(b) If $\vec{a}$ is constant vector, $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$ then, $\nabla(\vec{a}\cdot\vec{r}) = \vec{a}$

(c) If $\vec{a}$ is constant vector, $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$ then, $\nabla\cdot(\vec{a}\times\vec{r}) = \vec{a}$

(d) If $\vec{a}$ is constant vector, $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$ then, $\nabla\cdot(\vec{a}\times\vec{r}) = 0$ 31.

34. Which of the following is TRUE ?

(a) $\int_0^\infty x^3 e^{-4x} dx = \frac{3}{128}$ (b) $\int_0^\infty x^3 e^{-4x} dx = \frac{3}{32}$

(c) $\int_0^\infty x^{1/2} e^{-x^2} dx = \left|\left(\frac{3}{4}\right)\right.$ (d) $\int_0^\infty x^{1/2} e^{-x^2} dx = \frac{1}{2}\left|\left(\frac{3}{4}\right)\right.$

35. Which of the following is TRUE ?

(a) $\int_0^1 x^4\sqrt{(1-x^2)}\,dx = \frac{\pi}{16}$
(b) $\int_0^1 x^4\sqrt{(1-x^2)}\,dx = \frac{\pi}{32}$
(c) $\int_0^{\pi/2} \sin^5\theta\cos^4\theta\,d\theta = \frac{8}{315}$
(d) $\int_0^{\pi/2} \sin^5\theta\cos^4\theta\,d\theta = \frac{4}{315}$

36. Which of the following is TRUE ?

(a) $\int_0^1\int_y^1 e^{x^2}\,dx\,dy = \int_0^1\int_0^x e^{x^2}\,dy\,dx$
(b) $\int_0^1\int_y^1 e^{x^2}\,dx\,dy = \int_0^1\int_0^y e^{x^2}\,dy\,dx$
(c) $\int_0^1\int_{y^2}^y \frac{\sin y}{y}\,dx\,dy = \sin 1$
(d) $\int_0^1\int_{y^2}^y \frac{\sin y}{y}\,dx\,dy = 1-\sin 1$

37. The number of groups of order n (up to isomorphism) is

(a) finite for all values of n
(b) finite only for finitely many values of n
(c) finite for infinitely many values of n
(d) infinite for some values of n

38. Let $\sigma = (12)\,(345)$ and $\tau = (123456)$ be permutations in S_6, the group of permutations on six symbols, which of the following statement(s) is TRUE ?

(a) The subgroup $\langle\sigma\rangle$ and $\langle\tau\rangle$ are isomorphic to each other
(b) σ and τ are conjugate in S_6
(c) $\langle\sigma\rangle \cap \langle\tau\rangle$ is the trivial group
(d) σ and τ commute

39. Let G be an abelian group then choose the correct statement(s) :

(a) $f(x) = x^2$ be a homomorphism of G
(b) $f(x) = x^2$ be a automorphism of G
(c) $f(x) = x^p$, p is prime be a automorphism of G
(d) $f(x) = x^2$ be a automorphism of G if two does not divide $O(G)$

40. Let x be the set of all non-empty finite subsets of $\mathbb{N}$. Which of the following is/are an equivalence relation on x.

(a) ARB iff $\min A = \min B$
(b) ARB iff $A \cdot B$ have same numbers of elements
(c) ARB iff $A = B$
(d) ARB iff $A \cap B = \phi$

SECTION-C : NUMERICAL ANSWER TYPE (NAT's)

Q.41 to Q.50 : Carry 1 Mark each.

41. Let $I = \frac{1}{5}\int_P^Q \vec{F}\cdot d\vec{r}$ where P is the point (2, 1, 1) and Q is the point (3, 2, 2) and

$\vec{F} = y^2 z\hat{i} + 2xyz\,\hat{j} + (2z + xy^2)\hat{k}$ then I is ....

42. Let a, b are real number such that $\vec{F} = (x+z)\hat{i} + a(y+z)\,\hat{j} + b(x+y)\hat{k}$ is a conservative field then $a-b$ equals ....

43. Evaluate $\iint_S (3x\,dy\,dz + 2y\,dx\,dz - 5z\,dx\,dy)$ where S is a unit sphere ....

44. Evaluate $\iiint_E 2x\,dV$, where E is the region under the plane $2x+3y+z=6$ that lies in first octant ....

45. Let V the volume of the region that is below $z = 8 - x^2 - y^2$ above $z = -\sqrt{4x^2+4y^2}$ and inside $x^2+y^2=4$ then $\frac{V}{104\pi}$ ....

46. Evaluate $\frac{1}{14\pi}\int_0^1\int_0^{\sqrt{1-x^2}} \left(x^2+y^2\right)^{5/2} dydx$ ....

47. Let G be the group of all symmetries of the square. Then the number of conjugate classes in G is ....

48. The number of subgroups of the group $\mathbb{Z}_2 \oplus \mathbb{Z}_5$ is ....

49. The number of non trivial homomorphism from the group $\mathbb{Z}/12\mathbb{Z}$ to group $\mathbb{Z}/13\mathbb{Z}$ is ....

50. Let S be the boundary of the region $x^2+y^2 \le 4, 0 \le z \le 3$ oriented with unit normal pointing out words consider the vector field $F = \left(x^3 + \cos\left(y^2\right)\right)\hat{i} + yz\,\hat{j} + \left(3y^2 z + \cos(xy)\right)\hat{k}$ then $\frac{1}{30\pi}\iint_S F \cdot dS$ equals to ....

Q.51 to Q.60 : Carry 2 Marks each.

51. Let $\vec{F} = x^2\hat{i} + z^2 y\,\hat{j} + y^2 z\hat{k}$. Let the line integral of $\vec{F}$ around the unit sphere ....

52. Let $a\hat{i} + b\,\hat{j} + c\hat{k}$ be the direction from the point (1, 1, 0) which gives the greatest rate of increase of the function $\phi = (x+3y)^2 + (2y-z)^2$ then $a+b+c$ is ....

53. Evaluate $\frac{1}{4}\int_S F \cdot ds$ where $\vec{F} = 2y\,\hat{j} + z\hat{k}$ and S is the surface $x^2+y^2=4$ in the first two octant bounded by the planes $z=0, z=5$ and $y=0$ ....

54. Let surface area of the part of the sphere $x^2+y^2+z^2=36$ inside the cylinder $x^2+y^2=6y$ and above xy plane is $72\left(\frac{\pi}{2}-k\right)$ then k is ....

55. Let volume with in the cylinder $r = 4\cos\theta$ bounded by the sphere $r^2+z^2=16$ and below by the plane $z=0$ is $\frac{64}{9}(3\pi - c)$ then what is c ....

56. Let $f = (0,\infty) \to \mathbb{R}$ be a continuous function such that $\int_0^x f(t)\,dt = -2 + \frac{x^2}{2} + 4x\sin 2x + 3\cos 2x$, then the value of $\frac{1}{\pi} f\left(\frac{\pi}{4}\right)$ is ....

57. Let $G = \{e, x, x^2, x^3, y, xy, x^2y, x^3y\}$ with order $O(x) = 4, O(y) = 2$ and $xy = yx^3$, then the number of elements in the center of the group G is equal to ....

58. Let G be a group of order 30. Let H and K be normal subgroups of orders 2 and 5 respectively. Then the order of the group G/HK is ....

59. The number of homomorphism from $\mathbb{Q}_8$ to $\mathbb{Z}_8$ is ....

60. The number of group of order 12 up to isomorphic is ....

****** **END** ******

Space for rough work

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IIT-JAM-MATHEMATICS **08-01-2017**

TEST SERIES - 2

Integral Calculus + Group Theory + Vector Calculus

ANSWER KEY

SECTION-A

1. (b)	2. (d)	3. (c)	4. (d)	5. (c)
6. (a)	7. (a)	8. (d)	9. (c)	10. (c)
11. (a)	12. (a)	13. (a)	14. (d)	15. (a)
16. (b)	17. (b)	18. (c)	19. (b)	20. (b)
21. (c)	22. (d)	23. (d)	24. (d)	25. (d)
26. (c)	27. (d)	28. (a)	29. (c)	30. (b)

SECTION-B

31. (a, b)	32. (b, c)	33. (a, c)	34. (a, d)	35. (b, c)
36. (a, d)	37. (a, c)	38. (a, c)	39. (a, d)	40. (a, b, c)

SECTION-C

41. (5)	42. (0)	43. (0)	44. (9)	45. (0.333)
46. (1)	47. (5)	48. (4)	49. (0)	50. (3)
51. (0)	52. (7)	53. (5)	54. (1)	55. (4)
56. (0.25)	57. (2)	58. (3)	59. (4)	60. (5)