IIT-JAM MATHEMATICS
Test : Modern Algebra

Time : 60 Minutes   **Date : 08-10-2017**
                                                          M.M. : 45

INSTRUCTION:

1.   **Section-A** contains **10 Multiple Choice Questions (MCQ)**. Each question has 4 choices (a), (b), (c) and (d), for its answer, out of which ONLY ONE is correct. From **Q.1 to Q.10** carries **2 Marks** each. For each incorrect answered **0.5 mark** will be deducted.

2.   **Section-B** contains **5 Multiple Select Questions (MSQ)**. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE or MORE than ONE is/are correct. **Q.11 to Q. 15** for each correct answer you will be awarded **3 marks**. There is no negative marking in this section.

3.   **Section-C** contains **5 Numerical Answer Type (NAT)** questions. **Q.16 to Q.20** carries **2 Marks** each. There is no negative marking in this section.

SECTION-A [Multiple Choice Questions]

1.   Assume that the equation $xyz = 1$ holds in a group. Then

   A) $yzx = 1$ B) $yxz = 1$ C) $xzy = 1$ D) $None$

2.   If every element of a group G is its own inverse, then G is

   A) Cyclic group   B) Finite group

   C) Infinite group   D) Abelian group

3.   A Relation R is defined on the set of integers as aRb if and only if a^2 and b^2 is not prime to each other, then Relation does not satisfy the property

   A) Reflexive   B) Symmetric   C) Transitive   D) None

4.   If p is a prime number and G is a non-abelian group of order p^3 then the centre of has exactly

   A) $(p + 1)$ elements

   B) p^2 elements

   C) p elements

   D) $(p - 1)$ elements

5.   Let G be a Group and let H and K be two subgroup of G. If both H and K have 12 elements then which of the following numbers cannot be the cardinality of the set

$$HK = \{hk \; ; h \; \epsilon \; H, k \; \epsilon \; K\} ?$$

   A) 72   B) 60   C) 48   D) 36

6.   In U (40) , the cyclic subgroup of order 4 are

   A) 4   B) only one

   C) at most equal to the order of the group

   D) exactly two

7.   The elements of order 5 in S_7 are

   A) 120   B) 21   C) 504   D) 24

8.   Consider the following statements:

Statement A: All cyclic group are Abelian.

Statement B: The order of a cyclic group is same as the order of its generator.

Choose the correct option,

   A) Both A and B are false

   B) A is true, B is false

   C) B is true, A is false

   D) A and B are true

9. If $14 = 172 \ (mod \ x)$, then x can take the value

   A) 38   B) 54   C) 66   D) 79

10. The order of permutation
$$\rho = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 8 & 9 & 6 & 5 & 4 & 2 & 3 & 1 \end{pmatrix} \text{ is}$$

   A) 3   B) 9   C) 6   D) 4

SECTION-B [Multiple Select Questions]

11. An example of an infinite group in which every element has finite order is

   A) Non- singular 2×2 matrices with integer entries

   B) $\left(\frac{\mathbb{Q}}{\mathbb{Z}}, +\right) = \{\, r + \mathbb{Z} : r \in \mathbb{Q} \,\}$ Under addition defined as
   $$\big((r_1 + \mathbb{Z}) + (r_2 + \mathbb{Z})\big) = (r_1 + r_2) + \mathbb{Z}$$

   C) The invertible elements in $\mathbb{Z}$ under addition

   D) The Quaternion group

12. Let a_n denote the number of those permutations σ on $\{1, 2, \dots \dots \dots \ n\}$ such that σ is a product of exactly two disjoint cycles. Then

   A) $a_5 = 5$ B) $a_4 = 14$ C) $a_5 = 40$ D) $a_4 = 11$

13. Let G be a finite group of order n. Pick each correct statements from below-

A) if d divides n , there exist a subgroup of G of order d.

B) if d divides n , there exist an element of order d in G.

C) if every proper subgroup of G is cyclic, then G is cyclic.

D) None of these

14. Which of the following prime satisfy the congruence
$$a^{24} = 6a + 2$$

A) 41   B) 47   C) 67   D) 83

15. The following table defines a cyclic group

	A	B	C	D
A	C	A	D	B
B	A	B	C	D
C	D	C	B	A
D	B	D	A	C

The generators are

A) D   B) C   C) B   D) A

16. If $7x = 13 \ (mod \ 11)$, then the value of x is ------------------.

17. Let $o \ (G) = 24$ and G is cyclic. If $a \in G$ such that $a^8 \neq e, a^{12} \neq e$ then order of a is-------------------.

18. In A_4 number of elements satisfying $x^4 = e$ are -------------.

19. The last two digit of the number $37^{37^{21}}$ is -----------------.

20. The number of cyclic subgroup of D_8 are----------------.

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ANSWER KEY

SECTION-A [Multiple Choice Questions]

1. (a)	**2.** (d)	**3.** (a, c)	**4.** (c)
5. (b)	**6.** (a)	**7.** (c)	**8.** (d)
9. (d)	**10.** (c)		

SECTION-B [Multiple Select Questions]

11. (b)	**12.** (d)	**13.** (d)	**14.** (a, c)
15. (a, d)			

SECTION-C [Numerical Answer Type]

16. (5)	**17.** (24)	**18.** (4)	**19.** (17)
20. (12)			