Target IIT-JAM-2018

Test Series-4

Full Length Test Series-I

Booklet Code: D

Duration: 3:00 Hours | **MATHEMATICS-MA** | **Date: 14-01-2018**

Maximum Marks: 100

Read the following instructions carefully:

1. Attempt all the questions.
2. **Section-A** contains **30** Multiple Choice Questions (MCQ). Each question has 4 choices (a), (b), (c) and (d), for its answer, out of which **ONLY ONE** is correct. From **Q.1 to Q.10** carries 1 Marks and **Q.11 to Q.30** carries 2 Marks each.
3. **Section-B** contains **10** Multiple Select Questions(MSQ). Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which **ONE or MORE than ONE** is/are correct. For each correct answer you will be awarded **2 marks.**
4. Section**-C** contains **20** Numerical Answer Type (NAT) questions. From **Q.41 to Q.50** carries **1 Mark** each and **Q.51 to Q.60** carries **2 Marks** each. For each NAT type question, the value of answer in between 0 to 9.
5. In all sections, questions not attempted will result in zero mark. In Section–A (MCQ), wrong answer will result in negative marks. For all **1 mark** questions, **1/3 marks** will be deducted for each wrong answer. For all **2 marks** questions, **2/3 marks** will be deducted for each wrong answer. In Section–B (MSQ),there is no negative and no partial marking provisions. There is no negative marking in Section –C (NAT) as well.

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SECTION-A

[Multiple Choice Questions (MCQ)]

Q. 1 – Q. 10 carry one mark each.

1. Let a and a^2 both are the generations of a group G then order of G is

(a) a prime number (b) a odd number

(c) a perfect square (d) cannot say anything

2. The limit, $\lim \dfrac{n^n}{(n+1)(n+2)...(n+n)}$ is

(a) 0 (b) e (c) 1 (d) $\dfrac{e}{2}$

3. The orthogonal trajectories of the family $x^2 - y^2 = c_1$ are given by

(a) $x^2 + y^2 = c_2$ (b) $xy = c_2$

(c) $y = c_2$ (d) $x = c_2$

4. $\text{curl}\left[\nabla \cdot \left\{ r\nabla\left(\dfrac{1}{r^3}\right)\right\}\right]$ is equal to

(a) $\dfrac{2}{r^4}$ (b) $\dfrac{3}{r^4}$ (c) 0 (d) none

5. The function $f(x)$ defined by, $f(x) = \begin{cases} ax+b; & x \geq 1 \\ x^2+3x+3; & x \leq 1 \end{cases}$ is differentiable

(a) for a unique value of a and infinitely many values of b.
(b) for a unique value of b and infinitely many values of a
(c) for infinitely many values of a and b
(d) None of the above

6. Find the area enclosed between the curves $x = a\cos^3\theta, y = a\sin^3\theta$ in the first quadrant

(a) $\dfrac{32}{3}\pi a^2$ (b) πa^2

(c) $\dfrac{3}{32}\pi a^2$ (d) None of these

7. If $\int_{\sin x}^{1} t^2(f(t))dt = (1-\sin x)$, then $f\left(\dfrac{1}{\sqrt{3}}\right)$ is

(a) $\dfrac{1}{3}$ (b) $\dfrac{1}{\sqrt{3}}$ (c) 3 (d) $\sqrt{3}$

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8. Let A and B be square matrices such that AB = I, the identity matrix. Then zero is an eigen value of

(a) A but not of B (b) B but not of A

(c) Both A and B (d) Neither A nor B

9. Let G be a finite group then the possibility, that the numbers of elements of order three in G is

(a) 1 (b) 11 (c) 121 (d) 144

10. The differentiable equation $(3a^2x^2 + by\cos x)dx + (2\sin x - 4ay^3)dy = 0$ is exact if

(a) $a = 3, b = 2$ (b) $a = 2, b = 3$

(c) $a = 3, b = 4$ (d) $a = 2, b = 5$

Q. 11 – Q. 30 carry two marks each.

11. For $S \subseteq \mathbb{R}$, let ∂S denote the boundary of S and $\bar{S}$ denote, the closure of S. Then which one of the following is false?

(a) S is closed iff $S = \bar{S}$ (b) S is closed iff $\partial(S) \subseteq S$

(c) $\partial(\bar{S}) \subseteq \partial(S)$ (d) $\partial(\bar{S}) = \partial(S)$

12. Let G be a group of order '40' and homomorphic image of G is a cyclic group of order '8', then from the given information one can must be conclude that

(a) G has only normal subgroup of order 20

(b) G has normal subgroup of orders 4, 20

(c) G has normal subgroup of orders 5, 10, 20, 40

(d) Nothing can be concluded from the given information

13. A matrix M has eigenvalues 1 and 4 with corresponding eigenvectors $(1, -1)^T$ and $(2, 1)^T$, respectively. Then M is

(a) $\begin{pmatrix} -4 & -8 \\ 5 & 9 \end{pmatrix}$ (b) $\begin{pmatrix} 9 & -8 \\ 5 & -4 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$ (d) $\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$

14. $f(x,y) = \begin{cases} \dfrac{x^2 + y^2}{x - y} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ have

(a) continuity at (0, 0) and posses partial derivative at (0, 0)

(b) discontinuous but posses partial derivative at (0, 0)

(c) does not posses partial derivative at (0, 0)

(d) none of these

15. $\oint_c \dfrac{xdy - ydx}{x^2 + 4y^2}$ over any closed curve containing origin in anti-clockwise direction

(a) 0 (b) π (c) $-\pi$ (d) 1

16. Find $\iiint \frac{dxdydz}{(x+y+z+1)^3}$ throughout the volume bounded by the coordinate planes and the plane $x+y+z=1$

(a) $\frac{1}{2}\left(\log 2-\frac{5}{3}\right)$ (b) $\frac{1}{2}\left(\log 2-\frac{5}{8}\right)$

(c) $\frac{1}{2}\left(\log 2+\frac{5}{8}\right)$ (d) $\frac{1}{2}\left(\log 2+\frac{5}{3}\right)$

17. Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be two series, where $a_n=\frac{(-1)^n n}{2^n}$, $b_n=\frac{(-1)^n \log n}{n^2}$ for all $n\in\mathbb{N}$. Then

(a) both $\sum a_n$ and $\sum b_n$ are absolutely convergent

(b) $\sum b_n$ is absolutely convergent but $\sum a_n$ is conditionally convergent

(c) $\sum a_n$ is absolutely convergent $\sum b_n$ is conditionally convergent

(d) both $\sum a_n$ and $\sum b_n$ are conditionally convergent

18. Consider the ODE $y''=-4(y+y')$ with the boundary conditions that $y(x)=0$ at $x=\frac{1}{5}$. If we plotted the group of y as function of f for $x\geq 0$, then we can say that, the value of y

(a) first increases, then decreases to zero

(b) first decreases, then increases to zero

(c) has an extremum in the range $0<x<1$

(d) Insufficient data, cannot say anything about $y(x)$

19. Which of the following is uniformly continuous

(a) $\frac{\sin^2 x}{\sin x^2}$ on $(0,1)$ (b) $e^x \sin\frac{1}{x}$ on $(0,1)$

(c) $\cos x \sin\frac{1}{x}$ on $(0,1)$ (d) $e^{-x}\sin\frac{1}{x}$ on $(0,1)$

20. Let $y_1(x)$ and $y_2(x)$ be two linearly independent solutions of the differential equation

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+f(x)\cdot y=0$$

Let $W(x)=y_1(x)\cdot\frac{dy_2(x)}{dx}-y_2(x)\cdot\frac{dy_1(x)}{dx}$

If $W(0)=1$, then $W(1)$ is given by

(a) 1 (b) e^2 (c) $1/e$ (d) $1/e^2$

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21. Let $f(x,y) = x^5 \dfrac{e^{y/x}(x+y)}{(x^2+y^2)^{3/2}}$. Then $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y}$ equals

(a) $2f$ (b) $3f$ (c) $5f$ (d) $7f$

22. Which of the following statements is true for a square matrix A?

(a) If $A^2 = 0$, it necessarily implies that $A = 0$

(b) If A is real and orthogonal, its eigenvalues will always be real

(c) If A is hermitian, its diagonal entries are always real

(d) If A is anti-hermitian (skew-hermitian), its diagonal entries are always zero

23. $\int_c (y\hat{i} + z\hat{j} + x\hat{k}) \cdot d\vec{r}$, where c is intersection of $x^2 + y^2 + z^2 = 1$ and $x + y = 0$ traversed in clockwise direction when viewed in clockwise direction when viewed from (–1, –1, 0)

(a) $\sqrt{2}\,\pi$ (b) $-\sqrt{2}\,\pi$ (c) 2π (d) π

24. Which of the following must be true

(a) If G/H has an element of order '5' then G has an element of order 5

(b) If G is non-Abelian group then Aut(G) is not cyclic

(c) If G is non-Abelian group then inner Aut(G) is non-abelian

(d) A_5 has a normal subgroup of order 2

(Hint : $Inn(D_4) \approx K_4$)

25. Let $f(x,y) = \begin{cases} \dfrac{x}{|x|}\sqrt{x^2+y^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ then $f_x(0,0) + f_y(0,0)$ equals

(a) 1 (b) –1 (c) –2 (d) 0

26. Let A is the set of decimal functions between 0 and 1 whose only digits are zeros and ones. Then

(a) $\sup(A) = 1,\ \inf(A) = 0$ (b) $\sup(A) = \dfrac{1}{9},\ \inf(A) = 0$

(c) $\sup(A) = 1,\ \inf(A) = \dfrac{1}{10}$ (d) $\sup(A) = \dfrac{1}{9},\ \inf(A) = \dfrac{1}{10}$

27. The integral $\int_{x=-1}^{1} \int_{y=1+x}^{1} \cos(x+y) e^{y-x}\, dy\, dx$ is equivalent to

(a) $\dfrac{1}{2}\int_{u=-1}^{1} \int_{v=-u}^{1} e^v \cos u\, dv\, du$ (b) $\dfrac{1}{2}\int_{u=-1}^{1} \int_{v=-1}^{1} e^v \cos u\, dv\, du$

(c) $\dfrac{1}{2}\int_{u=-1}^{1} \int_{v=u}^{1} e^v \cos u\, dv\, du$ (d) None

28. $\iint_S (x^3dydz + x^2ydzdx + x^2zdxdy)$, where S is the sphere $x^2 + y^2 + z^2 = 1$

(a) $\frac{2\pi}{3}$ (b) $\frac{4\pi}{3}$ (c) $-\frac{4\pi}{3}$ (d) none

29. Let P be a matrix of order $m \times n$ and Q be a matrix of order $n \times p, n \neq p$. If $rank(P) = n$ and $rank(Q) = p$, then $rank(PQ)$ is

(a) n (b) p (c) np (d) $n + p$

30. The group $G = \{\langle a, b \rangle a^5 = b^2 = e, ba = a^2b\}$ is isomorphic to

(a) $\mathbb{Z}_2$ (b) $\mathbb{Z}_2 \oplus \mathbb{Z}_5$ (c) D_5 (d) $\mathbb{Z}_2 \oplus \mathbb{Z}_2$

SECTION-B

[Multiple Select Questions (MSQ)]

Q. 31 – Q. 40 carry two marks each.

31. Let $y^{(n)} + a_1y^{(n-1)} + ... + a_ny = 0$ be a n^{th} order homogeneous linear differential equation. Then which of the following is/are true? ($a_1, a_2, ..., a_n$ are real constants and $y^{(n)}$ denotes the n^{th} order derivative)

(a) The solutions of the equation form an n-dimensional vector space over $\mathbb{C}$

(b) Let $\phi_1, \phi_2, ..., \phi_n$ be n linearly independent solutions, then $W(\phi_1, \phi_2, ..., \phi_n)(x) \neq 0 \nvdash x \in I$

(c) Any linear combination of solutions is also a solution of the equation

(d) The solutions do not form a vector space over $\mathbb{R}$

32. Let $\vec{F} = e^x \sin yz\hat{i} + ze^x \cos yz\hat{j} + ye^x \cos yz\hat{k}$ be a force field, then which of the following is / are true?

(a) Total work done along $C : x^2 + y^2 = 1$ is '0'

(b) Total work done along the straight line joining (0, 0, 0) to (0, 1, $\pi/2$) is 1

(c) The scalar potential for force field $\vec{F}$ is $\phi(x, y, z) = e^x \sin y^2$ s.t. $\vec{F} = grad\ \phi$

(d) $\vec{F}$ is irrotational

33. If Inn(G) denote the inner automorphism on G and D_n denote dihedral group of order 2n choose the correct statements.

(a) Inn (D_6) contains only two elements of order three

(b) Inn (D_4) contains only two elements of order four

(c) Inn (D_6) contains only two elements of order six

(d) Inn (D_6) contains only three elements of order two

34. Let $T : R^n \to R^n$ be linear transformation and let A be the standard matrix for T, then select the correct statement

(a) T maps R^n onto R^m iff the columns of A span R^m

(b) T is one-one iff the columns of A are L.I.

(c) T is one-one if the equation T(x) = 0 has only trivial solution

(d) T is neither one-one nor onto

35. Which of the following are true?

(a) $\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2}}+\frac{1}{\sqrt{n^2-1}}+\ldots+\frac{1}{\sqrt{n^2-(n-1)^2}}\right)=\frac{\pi}{2}$

(b) $\lim_{n\to\infty}\frac{1}{n}\left[\frac{n+1}{n^2+1}+\frac{n+2}{n^2+2^2}+\ldots+\frac{1}{n}\right]=\frac{\pi}{4}+\ln 2$

(c) $\lim_{n\to\infty}\left(\frac{1}{\sqrt{4n^2-1}}+\frac{1}{\sqrt{4n^2-2^2}}+\ldots+\frac{1}{\sqrt{4n^2-2^2}}\right)=\frac{\pi}{6}$

(d) $\lim_{n\to\infty}\frac{1}{n}\left(\sin^2\frac{\pi}{2n}+\sin^2\frac{2\pi}{2n}+\ldots+\sin^2\frac{n\pi}{2n}\right)=\frac{1}{2}$

36. Which of the following statement is (are) true

(a) Each bounded sequence of real numbers has a convergent subsequence

(b) Each sequence has a convergent subsequence

(c) Each sequence has a monotone subsequence

(d) Each monotone sequence has a convergent subsequence

37. $f(x)=\int_{-1}^{x}(t^2+2t)(t^2-1)dt$

(a) Increases in $(-\infty,2)\cup(-1,0)\cup(1,\infty)$ (b) Increases in $(2,\infty)$

(c) Decreases in $[-2,-1]\cup[0,1]$ (d) Decreases in $(-\infty,2]$

38. Which of the following are conservative vector field functions

(a) $y^2\hat{i}+(2xy+e^{3z})\hat{j}+3ye^{3z}\hat{k}$ (b) $y\hat{i}+x\hat{j}+xyz\hat{k}$

(c) $\frac{-y\hat{i}+x\hat{j}}{x^2+y^2}$ (d) $yz\hat{i}+xz\hat{j}+xy\hat{k}$

39. Consider the differential equation, $\frac{d^2y}{dx^2}+\lambda y=0, y(0)=0, y(\pi)=0$. Then,

(a) The differential equation has the unique solution for $\lambda\le 0$

(b) The differential equation has the unique solution for $\lambda\in\mathbb{R}$

(c) The differential equation has infinitely many solutions for $\lambda>0$

(d) The differential equation has no solution

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40. Let n be a positive integer and V be an $(n+1)$–dimensional vector space over $\mathbb{R}$. If $\{e_1, e_2, ..., e_{n+1}\}$ is a basis of V and $T: V \to V$ is the linear transformation satisfying

$T(e_i) = e_{i+1}$ for $i = 1, 2, ..., n$ and $T(e_{n+1}) = 0$

Then

(a) trace of T is non-zero

(b) rank of T is n

(c) nullity of T is 1

(d) $T^n = ToToTo...^oT$ (n times) is the zero map

SECTION-C

[Numerical Answer Type (NAT)]

Q. 41 – Q. 50 carry one mark each.

41. Number of cyclic subgroups of order '7' in $\mathbb{Z}_7 \oplus \mathbb{Z}_7 \oplus \mathbb{Z}_7$ _____

42. The radius of convergence of the power series $\sum_{n=0}^{\infty} 2^{2n} x^{n^2}$ is _____

43. Let A be a 5 × 5 matrix with real entries such that the sum of the entries in each row of A is 1. Then the sum of all the entries in A^3 is _____

44. Consider the differential equation $(3xy + y^2)dx + (x^2 + xy)dy = 0$, if the integrating factor is given of the form $x^\alpha y^\beta$, then $\alpha + \beta$ is _____

45. The area of region bounded by $x + 2y^2 = 0$ and $3y^2 + x = 1$ _____

46. The $\left[\lim_{x \to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}\right]$ = _____

where [.] denotes greatest integer value of '0'.

47. The total area bounded by the lines $y = |x|, |x| = 1$ and $y = 0$ is _____

48. $T: R^4 \to R^3$ such that $T(x, y, z, w) = (x - y + z + w, x + 2z - w, x + y + 3z - 3w)$, then the dim of its range is _____

49. The set of all limit points of the set $\left\{\frac{2}{x+1} : x \in (-1, 1)\right\}$ in $\mathbb{R}$ is _____

50. $\int_S xyz dS$ over the portion of $x + y + z = a$, $a > 0$, lying in first octant is equal to $\frac{1}{k\sqrt{3}}$ then k is equal to _____

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Q. 51 – Q. 60 carry two marks each.

51. Limit of the sequence (x_n), where $x_1 = 1,\ x_2 = 2$ and $x_n = \frac{4}{5}x_{n-1} + \frac{1}{5}x_{n-2} \forall\, n \geq 3$ is _____

52. A 3 × 3 matrix has elements such that its trace and determinant are 11 and 36 respectively. All the eigenvalues of the matrix are known to be positive. Then the largest eigenvalues of matrix is _____

53. Let G be a group and $x, y \in G$ such that order of x is '5' and $ab = b^{-1}a$. If $y \neq e$ then order of y is _____

54. Consider the differential equation $\frac{d^2x}{dx^2} + 2\frac{dx}{dt} + x = 0$ with initial conditions $x(0) = 0$ and $x'(0) = 1$, then solution $x(t)$ attains its maximum value when t is _____

55. The area enclosed by the parabola $y^2 = 2x$ and two tangents to the parabola from point (–2, 0) _____

56. Let $f(x) = |(x^2 - x - 20)|$ for all $x \in \mathbb{R}$. The total number of points of $\mathbb{R}$ at which f attains its local extremum (minimum or maximum) is _____

57. Let $\phi : \mathbb{Z} \oplus \mathbb{Z} \to \mathbb{Z}$ be an homomorphism, given that $\phi(3,2) = 3$ and $\phi(2,1) = 2$ then $\phi(4,4)$ is _____

58. Let $f(x,y) = \begin{cases} \frac{xy(x^2 - y^2)}{x^2 + y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ then $f_{xy}(0,0) + f_{yx}(0,0)$ is equal to _____

59. $\oint_S \vec{F} \cdot \hat{n} dS$, where S is the entire surface of the region above xy plane bounded by the cone $z^2 = x^2 + y^2$ and the plane $z = 3$ if $\vec{F} = 4xz\hat{i} + xyz^2\hat{j} + 3z\hat{k}$, is equal to $k\pi$ then k is equal to _____

60. The value of particular integral of $(D^2 + 4)y = x^4$ at $x = 2$ is _____

END OF THE QUESTION PAPER

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SPACE FOR ROUGH WORK

IIT-JAM MATHEMATICS

TEST SERIES - 4

(Full Length Test Series - 1)

Time : 3 Hours **Date : 14-01-2018**

M.M. : 100

ANSWER KEY

SECTION-A

[Multiple Choice Questions (MCQ)]

1. ()	**2.** ()	**3.** ()	**4.** ()	**5.** ()
6. ()	**7.** ()	**8.** ()	**9.** ()	**10.** ()
11. ()	**12.** ()	**13.** ()	**14.** ()	**15.** ()
16. ()	**17.** ()	**18.** ()	**19.** (a)	**20.** ()
21. ()	**22.** ()	**23.** ()	**24.** ()	**25.** ()
26. ()	**27.** ()	**28.** ()	**29.** ()	**30.** ()

SECTION-B

[Multiple Select Questions (MSQ)]

31. ()	**32.** (a, b, c, d)	**33.** ()	**34.** ()
35. (a, b, c, d)	**36.** ()	**37.** (a, c)	**38.** (a, c, d)
39. ()	**40.** (zero)		

SECTION-C

[Numerical Answer Type (NAT)]

41. ()	**42.** ()	**43.** ()	**44.** ()	**45.** ()
46. ()	**47.** ()	**48.** ()	**49.** ()	**50.** ()
51. ()	**52.** ()	**53.** ()	**54.** ()	**55.** ()
56. ()	**57.** ()	**58.** ()	**59.** ()	**60.** ()