IIT-JAM-2015 (MATHEMATICS)

SECTION – A : MCQ

1. Suppose N is a normal subgroup of a group G. Which one of the following is true?
(A) If G is an infinite group then G/N is an infinite group
(B) If G is a non-abelian group then G/N is a non abelian group
(C) If G is a cyclic group then G/N is an abelian group
(D) If G is an abelian group then G/N is a cyclic group
(1) A (2) B (3) C (4) D

2. The volume of the portion of the solid cylinder $x^2 + y^2 \le 2$ bounded above by the surface $z = x^2 + y^2$ and bounded below by the xy-plane is
(A) π (B) 2π (C) 3π (D) 4π
(1) A (2) B (3) C (4) D

3. Let S be a nonempty subset of $\mathbb{R}$. If S is a finite union of disjoint bounded intervals, then which one of the following is true?
(A) If S is not compact, then sup S $\notin$ S inf S $\notin$ S
(B) Even if sup S $\in$ S and inf S $\in$ S, S need not be compact
(C) If sup S $\in$ S and inf S $\in$ S, then S is compact
(D) Even if S is compact, it is not necessary that sup S $\in$ S and inf S $\in$ S
(1) A (2) B (3) C (4) D

4. Let $\{x_n\}$ be a convergent sequence of real numbers. If $x_1 > \pi + \sqrt{2}$ and $x_{n+1} = \pi + \sqrt{x_n - \pi}$ for $n \ge 1$, then which one of the following is the limit of this sequence?
(A) $\pi + 1$ (B) $\pi + \sqrt{2}$ (C) π (D) $\pi + \sqrt{\pi}$
(1) A (2) B (3) C (4) D

5. Let a, b, c, d be distinct non-zero real numbers with $a + b = c + d$. Then an eigenvalue of the matrix

$$\begin{bmatrix} a & b & 1 \\ c & d & 1 \\ 1 & -1 & 0 \end{bmatrix} \text{ is}$$

(A) $a + c$ (B) $a + b$ (C) $a - b$ (D) $b - d$
(1) A (2) B (3) C (4) D

6. Let A be a nonempty subset of $\mathbb{R}$. Let I(A) denote the set of interior points of A. Then I(A) can be

(A) empty
(B) singleton
(C) a finite set containing more than one element
(D) countable but not finite

(1) A (2) B (3) C (4) D

7. Let $y(x) = u(x)\sin x + v(x)\cos x$ be a solution of the different equation $y'' + y = \sec x$. Then $u(x)$ is

(A) $\ln|\cos x| + c$
(B) $-x + c$
(C) $x + c$
(D) $\ln|\sec x| + c$

(1) A (2) B (3) C (4) D

8. An integrating factor of the differential equation $\frac{dy}{dx} = \frac{2xy^2 + y}{x - 2y^3}$ is

(A) $\frac{1}{y}$
(B) $\frac{1}{y^2}$
(C) y
(D) y^2

(1) A (2) B (3) C (4) D

9. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f(0) = 0$. If for all $x \in \mathbb{R}, 1 < f'(x) < 2,$ then which one of the following statements is true on $(0, \infty)$?

(A) f is unbounded
(B) f is increasing and bounded
(C) f has at least one zero
(D) f is periodic

(1) A (2) B (3) C (4) D

10. If an integral curve of the differential equation $(y - x)\frac{dy}{dx} = 1$ passes through (0, 0) and $(\alpha, 1)$, then α is equal to

(A) $2 - e^{-1}$
(B) $1 - e^{-1}$
(C) e^{-1}
(D) $1 + e$

(1) A (2) B (3) C (4) D

11. Let S be the bounded surface of the cylinder $x^2 + y^2 = 1$ cut by the planes $z = 0$ and $z = 1 + x$. Then the value of the surface integral $\iint_S 3z^2 d\sigma$ is equal to

(A) $\int_0^{2\pi} (1 + \cos\theta)^3 d\theta$
(B) $\int_0^{2\pi} \sin\theta\cos\theta(1 + \cos\theta)^2 d\theta$
(C) $\int_0^{2\pi} (1 + 2\cos\theta)^3 d\theta$
(D) $\int_0^{2\pi} \sin\theta\cos\theta(1 + 2\cos\theta)^2 d\theta$

(1) A (2) B (3) C (4) D

12. Let $P_2(\mathbb{R})$ be the vector space of polynomials in x of degree at most 2 with real coefficients. Let $M_2(\mathbb{R})$ be the vector space of 2×2 real matrices. If a linear transformation $T : P_2(\mathbb{R}) \to M_2(\mathbb{R})$ is defined as $T(f) = \begin{bmatrix} f(0)-f(2) & 0 \\ 0 & f(1) \end{bmatrix}$ then

(A) T is one-one but not onto
(B) T is onto but not one-one
(C) Range (T) = span $\left\{ \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} \right\}$
(D) Null (T) = span $\{x^2 - 2x, 1 - x\}$

(1) A (2) B (3) C (4) D

13. Let $S = \bigcap_{n=1}^{\infty} \left(\left[0, \frac{1}{2\pi+1} \right] \cup \left[\frac{1}{2n}, 1 \right] \right)$. Which one of the following statements is FALSE?

(A) There exist sequences $\{a_n\}$ and $\{b_n\}$ in [0, 1] such that $S = [0,1] \setminus \bigcup_{n=1}^{\infty} (a_n, b_n)$
(B) [0, 1]\S is an open set
(C) If A is an infinite subset of S, then A has a limit point
(D) There exists an infinite subset of S having no limit points

(1) A (2) B (3) C (4) D

14. The limit $\lim_{x \to 0+} \frac{1}{\sin^2 x} \int_{\frac{x}{2}}^{x} \sin^{-1} t \, dt$ is equal to

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{3}{8}$

(1) A (2) B (3) C (4) D

15. Let S_3 be the group of permutations of three distinct symbols. The direct sum $S_3 \oplus S_3$ has an element of order

(A) 4 (B) 6 (C) 9 (D) 18

(1) A (2) B (3) C (4) D

16. Let $B_1 = \{(1, 2), (2, -1)\}$ and $B_2 = \{(1, 0), (0, 1)\}$ be ordered bases of $\mathbb{R}^2$. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that $[T]_{B_1, B_2}$, the matrix of T with respect to B_1 and B_2, is $\begin{bmatrix} 4 & 3 \\ 2 & -4 \end{bmatrix}$, then T(5, 5) is equal to

(A) (–9, 8) (B) (9, 8) (C) (–15, –2) (D) (15, 2)

(1) A (2) B (3) C (4) D

17. Let G be a nonabelian group. Let $\alpha \in G$ have order 4 and let $\beta \in G$ have order 3. Then the order of the element $\alpha\beta$ in G.

(A) is 6
(B) is 12
(C) is of the form $12k$ for $k \geq 2$
(D) need not be finite

(1) A (2) B (3) C (4) D

18. Let $A=\begin{bmatrix} 0 & 1-i \\ -1-i & i \end{bmatrix}$ and $B=A^T\overline{A}$. Then

(A) an eigenvalue of B is purely imaginary
(B) an eigenvalue of A is zero
(C) all eigenvalues of B are real
(D) A has a non-zero real eigenvalue

(1) A (2) B (3) C (4) D

19. Suppose that the dependent variables z and w are functions of the independent variables x and y, defined by the equations $f(x,y,z,w)=0$ and $g(x,y,z,w)=0$, where $f_z g_w - f_w g_z = 1$. Which one of the following is correct?

(A) $z_x = f_w g_x - f_x g_w$
(B) $z_x = f_x g_w - f_w g_x$
(C) $z_x = f_z g_x - f_x g_z$
(D) $z_x = f_z g_w - f_z g_x$

(1) A (2) B (3) C (4) D

20. The orthogonal trajectories of the family of curves $y=c_1x^3$ are

(A) $2x^2+3y^2=c_2$
(B) $3x^2+y^2=c_2$
(C) $3x^2+2y^2=c_2$
(D) $x^2+3y^2=c_2$

(1) A (2) B (3) C (4) D

21. Which one of the following statements is true for the series $\sum_{n=1}^{\infty}(-1)^n\frac{(2n)!}{n^{2n}}$?

(A) The series converges conditionally but not absolutely
(B) The series converges absolutely
(C) The sequence of partial sums of the series is bounded but not convergent
(D) The sequence of partial sums of the series is unbounded

(1) A (2) B (3) C (4) D

22. Let G and H be nonempty subsets of $\mathbb{R}$, where G is connected and $G\cup H$ is not connected. Which one of the following statements is true for all such G and H?

(A) If $G\cap H=\varnothing$, then H is connected
(B) If $G\cap H=\varnothing$, then H is not connected
(C) If $G\cap H\neq\varnothing$, then H is connected
(D) If $G\cap H\neq\varnothing$, then H is not connected

(1) A (2) B (3) C (4) D

23. For $m,n\in\mathbb{N}$, define $f_{m,n}(x)=\begin{cases} x^m\sin\left(\frac{1}{x^n}\right), & x\neq 0 \\ 0, & x=0 \end{cases}$.

Then at $x=0, f_{m,n}$ is

(A) differentiable for each pair m, n with $m>n$
(B) differentiable for each pair m, n with $m<n$
(C) not differentiable for each pair m, n with $m>n$
(D) not differentiable for each pair m, n with $m<n$

(1) A (2) B (3) C (4) D

24. For what real values of x and y, does the integral $\int_x^y (6-t-t^2)dt$ attain its maximum?

(A) $x=-3, y=2$ (B) $x=2, y=3$

(C) $x=-2, y=2$ (D) $x=-3, y=4$

(1) A (2) B (3) C (4) D

25. Let $f:\{(x,y)\in\mathbb{R}^2 : x>0, y>0\}\to\mathbb{R}$ be given by

$$f(x,y)=x^{\frac{1}{3}}y^{\frac{-4}{3}}\tan^{-1}\left(\frac{y}{x}\right)+\frac{1}{\sqrt{x^2+y^2}}$$

Then the value of $g(x,y)=\dfrac{xf_x(x,y)+yf_y(x,y)}{f(x,y)}$

(A) changes with x but not with y (B) changes with y but not with x

(C) changes with x and also with y (D) neither changes with x nor with y

(1) A (2) B (3) C (4) D

26. Let $f:\mathbb{R}\to\mathbb{R}$ be a strictly increasing continuous function. If $\{a_n\}$ is a sequence in [0, 1], then the sequence $\{f(a_n)\}$ is

(A) increasing (B) bounded

(C) convergent (D) not necessarily bounded

(1) A (2) B (3) C (4) D

27. The area of the planar region bounded by the curves $x=6y^2-2$ and $x=2y^2$ is

(A) $\frac{\sqrt{2}}{3}$ (B) $\frac{2\sqrt{2}}{3}$ (C) $\frac{4\sqrt{2}}{3}$ (D) $\sqrt{2}$

(1) A (2) B (3) C (4) D

28. If $y(t)$ is a solution of the differential equation $y''+4y=2e^t$, then $\lim_{t\to\infty} e^{-t}y(t)$ is equal to

(A) $\frac{2}{3}$ (B) $\frac{2}{5}$ (C) $\frac{2}{7}$ (D) $\frac{2}{9}$

(1) A (2) B (3) C (4) D

29. The sequence $\left\{\cos\left(\frac{1}{2}\tan^{-1}\left(-\frac{n}{2}\right)^n\right)\right\}$ is

(A) monotone and convergent (B) monotone but not convergent

(C) convergent but not monotone (D) neither monotone nor convergent

(1) A (2) B (3) C (4) D

30. For $n\geq 2$, let $f_n:\mathbb{R}\to\mathbb{R}$ be given by $f_n(x)=x^n\sin x$. Then at $x=0, f_n$ has a

(A) local maximum if n is even (B) local maximum if n is odd

(C) local minimum if n is even (D) local minimum if n is odd

(1) A (2) B (3) C (4) D

SECTION – B : MSQ

1. Let $f:\mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y)=\begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$

At (0, 0),

(A) f is not continuous
(B) f is continuous, and both f_x and f_y exist
(C) f is differentiable
(D) f_x and f_y exist but f is not differentiable

(1) A (2) B (3) C (4) D

2. Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined by $f(x)=\int_{-5}^{x}(t-1)^3\,dt$. In which of the following interval(s), f takes the value 1?

(A) [–6, 0] (B) [–2, 4] (C) [2, 8] (D) [6, 12]

(1) A (2) B (3) C (4) D

3. Let $f, g:[0,1]\to[0,1]$ be functions. Let $R(f)$ and $R(g)$ be the ranges of f and g, respectively. Which of the following statements is (are) true?

(A) If $f(x)\le g(x)$ for all $x\in[0,1]$, then $\sup R(f)\le \inf R(g)$
(B) If $f(x)\le g(x)$ for some $x\in[0,1]$, then $\inf R(f)\le \sup R(g)$
(C) If $f(x)\le g(y)$ for some $x,y\in[0,1]$, then $\inf R(f)\le \sup R(g)$
(D) If $f(x)\le g(y)$ for all $x,y\in[0,1]$, then $\sup R(f)\le \inf R(g)$

(1) A (2) B (3) C (4) D

4. Let $f:(-1,1)\to\mathbb{R}$ be the function defined by $f(x)=x^2e^{1/(1-x^2)}$. Then

(A) f is decreasing in (–1, 0)
(B) f is increasing in (0, 1)
(C) $f(x)=1$ has two solutions in (–1, 1)
(D) $f(x)=1$ has no solutions in (–1, 1)

(1) A (2) B (3) C (4) D

5. Which of the following conditions implies (imply) the convergence of a sequence $\{x_n\}$ of real numbers?

(A) Given $\varepsilon>0$ there exists an $n_0\in\mathbb{N}$ such that for all $n\ge n_0, |x_{n+1}-x_n|<\varepsilon$
(B) Given $\varepsilon>0$ there exists an $n_0\in\mathbb{N}$ such that for all $n\ge n_0, \dfrac{1}{(n+1)^2}|x_{n+1}-x_n|<\varepsilon$
(C) Given $\varepsilon>0$ there exists an $n_0\in\mathbb{N}$ such that for all $n\ge n_0, (n+1)^2|x_{n+1}-x_n|<\varepsilon$
(D) Given $\varepsilon>0$ there exists an $n_0\in\mathbb{N}$ such that for all m, n with $m>n\ge n_0, |x_m-x_n|<\varepsilon$

(1) A (2) B (3) C (4) D

6. Which of the following statements is (are) true on the interval $\left(0, \frac{\pi}{2}\right)$?

(A) $\cos x < \cos(\sin x)$ (B) $\tan x < x$

(C) $\sqrt{1+x} < 1 + \frac{x}{2} - \frac{x^2}{8}$ (D) $\frac{1-x^2}{2} < \ln(2+x)$

(1) A (2) B (3) C (4) D

7. Which of the following statements is (are) true?

(A) $\mathbb{Z}_2 \oplus \mathbb{Z}_3$ is isomorphic to $\mathbb{Z}_6$ (B) $\mathbb{Z}_3 \oplus \mathbb{Z}_3$ is isomorphic to $\mathbb{Z}_9$

(C) $\mathbb{Z}_4 \oplus \mathbb{Z}_6$ is isomorphic to $\mathbb{Z}_{24}$ (D) $\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_5$ is isomorphic to $\mathbb{Z}_{30}$

(1) A (2) B (3) C (4) D

8. The initial value problem $y' = \sqrt{y}, y(0) = \alpha, \alpha \geq 0$ has

(A) at least two solutions if $\alpha = 0$ (B) no solution if $\alpha > 0$

(C) at least one solution if $\alpha > 0$ (D) a unique solution if $\alpha = 0$

(1) A (2) B (3) C (4) D

9. Let $\vec{F}$ be a vector field given by $\vec{F}(x,y,z) = -y\hat{i} + 2xy\hat{j} + z^3\hat{k}$, for $(x,y,z) \in \mathbb{R}^3$. If c is the curve of intersection of the surfaces $x^2 + y^2 = 1$ and $y + z = 2$, then which of the following is (are) equal to $\left|\int_C \vec{F} \cdot d\vec{r}\right|$?

(A) $\int_0^{2\pi} \int_0^1 (1 + 2r\sin\theta) r\, dr d\theta$ (B) $\int_0^{2\pi} \left(\frac{1}{2} + \frac{2}{3}\sin\theta\right) d\theta$

(C) $\int_0^{2\pi} \int_0^1 (1 + 2r\sin\theta)\, dr d\theta$ (D) $\int_0^{2\pi} (1 + \sin\theta)\, d\theta$

(1) A (2) B (3) C (4) D

10. Let V be the set of 2 × 2 matrices $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ with complex entries such that $a_{11} + a_{22} = 0$. Let W be the set of matrices in V with $a_{12} + \overline{a_{21}} = 0$. Then, under usual matrix addition and scalar multiplication, which of the following is (are) true?

(A) V is a vector space over $\mathbb{C}$ (B) W is a vector space over $\mathbb{C}$

(C) V is a vector space over $\mathbb{R}$ (D) W is a vector space over $\mathbb{R}$

(1) A (2) B (3) C (4) D

SECTION – C : NAT

1. If the set $\left\{ \begin{bmatrix} x & -x \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ x & x \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \right\}$ is linearly dependent in the vector space of all 2×2 matrices with real entries, then x is equal to ____________________

2. If $5^{2015} \equiv n \pmod{11}$ and $n \in \{0,1,2,3,4,5,6,7,8,9,10\}$, then n is equal to ______________

3. If the power series $\sum_{n=0}^{\infty} \frac{n!}{n^n} x^{2n}$ converges for $|x| < c$ and diverges for $|x| > c$, then the value of c, correct upto three decimal places, is ___________________

4. The number of distinct normal subgroups of S_3 is __________________

5. Let C be the straight line segment from $P(0,\pi)$ to $Q\left(4, \frac{\pi}{2}\right)$, in the xy-plane. Then the value of $\int_C e^x(\cos y\,dx - \sin y\,dy)$ is _______________

6. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = \begin{cases} \left(1+\frac{x}{y}\right)^2, & y \neq 0 \\ 0, & y = 0 \end{cases}$

If the directional derivative of f at (0, 0) exists along the direction $\cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}$, where $\sin\alpha \neq 0$, then the value of $\cot\alpha$ is _________________

7. Let $f : (0,1) \to \mathbb{R}$ be a continuously differentiable function such that f' has finitely mainly zeros in (0, 1) and f' changes sign at exactly two of these points. Then for any $y \in \mathbb{R}$, the maximum number of solutions to $f(x) = y$ in (0, 1) is _______________

8. Let $f : \mathbb{R}^3 \to \mathbb{R}$ be defined by $f(x,y,z) = \sin x + 2e^{\frac{y}{2}} + z^2$. The maximum rate of change of f at $\left(\frac{\pi}{4}, 0, 1\right)$, correct upto three decimal places, is _________________

9. Let S be the portion of the surface $z = \sqrt{16 - x^2}$ bounded by the planes $x = 0, x = 2, y = 0$, and $y = 3$. The surface area of S, correct upto three decimal places, is _________________

10. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \begin{cases} x^6 - 1, & x \in \mathbb{Q} \\ 1 - x^6, & x \notin \mathbb{Q} \end{cases}$.

The number of points at which f is continuous, is ________________

11. The coefficient of $\left(x - \frac{\pi}{4}\right)^3$ in the Taylor series expansion of the function $f(x) = 3\sin x \cos\left(x + \frac{\pi}{4}\right)$, $x \in \mathbb{R}$ about the point $\frac{\pi}{4}$, correct upto three decimal places, is _______________

12. Let $\mathbb{R}$ be the planar region bounded by the lines $x = 0, y = 0$ and the curve $x^2 + y^2 = 4$, in the first quadrant. Let C be the boundary of $\mathbb{R}$, oriented counter-clockwise. Then the value of $\oint_C x(1-y)dx + (x^2 - y^2)dy$ is ___________________

13. Let P and Q be two real matrices of size 4 × 6 and 5 × 4, respectively. If rank(Q) = 4 and rank(QP) = 2, then rank(P) is equal to ________________

14. Let ℓ be the length of the portion of the curve $x = x(y)$ between the lines $y = 1$ and $y = 3$, where $x(y)$ satisfies

$$\frac{dx}{dy} = \frac{\sqrt{1+y^2+y^4}}{y}, x(1) = 0$$

The value of ℓ, correct upto three decimal places, is __________________

15. If $\int_0^x (e^{-t^2} + \cos t)dt$ has the power series expansion $\sum_{n=1}^{\infty} a_n x^n$, then a_5, correct upto three decimal places, is equal to _____________________

16. Let $M_2(\mathbb{R})$ be the vector space of 2 × 2 real matrices. Let V be a subspace of $M_2(\mathbb{R})$ defined by

$$V = \left\{ A \in M_2(\mathbb{R}) : A\begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} A \right\}$$

Then the dimension of V is __________________

17. Suppose G is a cyclic group and $\sigma, \tau \in G$ are such that order (σ) = 12 and order (τ) = 21. Then the order of the smallest group containing σ and τ is __________________

18. The limit $\lim\limits_{x \to 0+} \frac{9}{x}\left(\frac{1}{\tan^{-1} x} - \frac{1}{x}\right)$ is equal to __________________

19. The limit $\lim\limits_{n \to \infty} \sum_{k=2}^{n} \frac{1}{k^3 - k}$ is equal to _________________

20. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = \begin{cases} \frac{y}{\sin y}, & y \neq 0 \\ 1, & y = 0 \end{cases}$

Then the integral $\frac{1}{\pi^2}\int_{x=0}^{1}\int_{y=\sin^{-1} x}^{\frac{\pi}{2}} f(x,y)dydx$ correct upto three decimal places, is ___________