# PHYSICS (PH)
# Joint Entrance Screening Test (JEST-2016)

## Part-A: 3-Mark Questions

1. The wavefunction of a hydrogen atom is given by the following superposition of energy eigen-functions $\psi_{nlm}(\vec{r})$ ($n, l, m$ are the usual quantum numbers);

$$\psi(\vec{r}) = \frac{\sqrt{2}}{\sqrt{7}}\psi_{100}(\vec{r}) - \frac{3}{\sqrt{14}}\psi_{210}(\vec{r}) + \frac{1}{\sqrt{14}}\psi_{322}(\vec{r})$$

The ratio of expectation value of the energy to the ground state energy and the expectation value of $L^2$ are, respectively:

(a) $\frac{229}{504}$ and $\frac{12\hbar^2}{7}$ (b) $\frac{101}{504}$ and $\frac{12\hbar^2}{7}$ (c) $\frac{101}{504}$ and $\hbar^2$ (d) $\frac{229}{504}$ and $\hbar^2$

2. An ideal gas with adiabatic exponent $\gamma$ undergoes a process in which its pressure $P$ is related to its volume $V$ by the relation $P = P_0 - \alpha V$, where $P_0$ and $\alpha$ are positive constants. The volume starts from being very close to zero and increases monotonically to $P_0/\alpha$. At what value of the volume during the process does the gas have maximum entropy?

(a) $\frac{P_0}{\alpha(1+\gamma)}$ (b) $\frac{\gamma P_0}{\alpha(1-\gamma)}$ (c) $\frac{\gamma P_0}{\alpha(1+\gamma)}$ (d) $\frac{P_0}{\alpha(1-\gamma)}$

3. The $H_2$ molecule has a reduced mass $M = 8.35\times10^{-28}$ kg and an equilibrium internuclear distance $R = 0.742 \times 10^{-10}$ m. The rotational energy in terms of the rotational quantum number $J$ is :

(a) $E_{\text{rot}}(J) = 7J(J-1)$ meV (b) $E_{\text{rot}}(J) = \frac{5}{2}J(J+1)$ meV

(c) $E_{\text{rot}}(J) = 7J(J+1)$ meV (d) $E_{\text{rot}}(J) = \frac{5}{2}J(J-1)$ meV



4. The Hamiltonian of a quantum particle of mass $m$ confined to a ring of unit radius is:

$$H = \frac{\hbar^2}{2m}\left(-i\frac{\partial}{\partial\theta} - \alpha\right)^2,$$

where $\theta$ is the angular coordinate, $\alpha$ is a constant. The energy eigenvalues and eigenfunctions of the particle are ($n$ is an integer):

(a) $\psi_n(\theta) = \frac{e^{in\theta}}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}(n-\alpha)^2$ (b) $\psi_n(\theta) = \frac{\sin(n\theta)}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}(n-\alpha)^2$

(c) $\psi_n(\theta) = \frac{\cos(n\theta)}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}(n-\alpha)^2$ (d) $\psi_n(\theta) = \frac{e^{in\theta}}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}(n+\alpha)^2$

5. Consider a quantum particle of mass $m$ in one dimension in an infinite potential well, i.e., $V(x) = 0$ for $-a/2 < x < a/2$, and $V(x) = \infty$ for $|x| \geq a/2$. A small perturbation, $V'(x) = 2\in|x|/a$, is added. The change in the ground state energy to $O(\varepsilon)$ is:

(a) $\frac{\varepsilon}{2\pi^2}(\pi^2-4)$ (b) $\frac{\varepsilon}{2\pi^2}(\pi^2+4)$ (c) $\frac{\varepsilon\pi^2}{2}(\pi^2+4)$ (d) $\frac{\varepsilon\pi^2}{2}(\pi^2-4)$

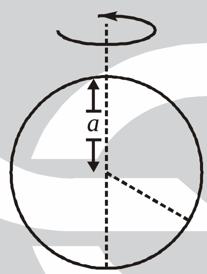
6. A spin-1 particle is in a state $|\psi\rangle$ described by the column matrix $(1/\sqrt{10})\{2, \sqrt{2}, 2i\}$ in the $S_z$ basis. What is the probability that a measurement of operator $S_z$ will yield the result $\hbar$ for the state $S_x|\psi\rangle$?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

7. The energy of a particle is given by $E=|p|+|q|$, where $p$ and $q$ are the generalized momentum and coordinate, respectively. All the states with $E \le E_0$ are equally probable and states with $E > E_0$ are inaccessible. The probability density of finding the particle at coordinate $q$, with $q>0$ is:

(a) $\frac{(E_0+q)}{E_0^2}$ (b) $\frac{q}{E_0^2}$ (c) $\frac{(E_0-q)}{E_0^2}$ (d) $\frac{1}{E_0}$

8. A hoop of radius $a$ rotates with constant angular velocity $\omega$ about the vertical axis as shown in the figure. A bead of mass $m$ can slide on the hoop without friction. If $g<\omega^2 a$, at what angle $\theta$ apart from 0 and $\pi$ is the bead stationary $\left(i.e., \frac{d\theta}{dt}=\frac{d^2\theta}{dt^2}=0\right)$?




(a) $\tan\theta=\pi g/\omega^2 a$ (b) $\sin\theta=g/\omega^2 a$ (c) $\cos\theta=g/\omega^2 a$ (d) $\tan\theta=g/\pi\omega^2 a$

9. A gas of $N$ molecules of mass $m$ is confined in a cube of volume $V=L^3$ at temperature $T$. The box is in a uniform gravitational field $-g\hat{z}$. Assume that the potential energy of a molecule is $U=mgz$, where $z\in[0, L]$ is the vertical coordinate inside the box. The pressure $P(z)$ at height $z$ is :

(a) $P(z)=\frac{N}{V}\frac{mgL}{2}\frac{\exp\left(-\frac{mg(z-L/2)}{k_BT}\right)}{\sinh\left(\frac{mgL}{2k_BT}\right)}$ (b) $P(z)=\frac{N}{V}\frac{mgL}{2}\frac{\exp\left(-\frac{mg(z-L/2)}{k_BT}\right)}{\cosh\left(\frac{mgL}{2k_BT}\right)}$

(c) $P(z)=\frac{k_BTN}{V}$ (d) $P(z)=\frac{N}{V}mgz$

10. A point charge $q$ of mass $m$ is released from rest at a distance $d$ from an infinite grounded conducting plane (ignore gravity). How long does it takes for the charge to hit the plane ?

(a) $\frac{\sqrt{2\pi^3\varepsilon_0 md^3}}{q}$ (b) $\frac{\sqrt{2\pi^3\varepsilon_0 md}}{q}$ (c) $\frac{\sqrt{\pi^3\varepsilon_0 md^3}}{q}$ (d) $\frac{\sqrt{\pi^3\varepsilon_0 md}}{q}$

11. The strength of magnetic field at the center of a regular hexagon with sides of length $a$ carrying a steady current $I$ is :

(a) $\frac{\mu_0 I}{\sqrt{3}\pi a}$ (b) $\frac{\sqrt{6}\mu_0 I}{\pi a}$ (c) $\frac{3\mu_0 I}{\pi a}$ (d) $\frac{\sqrt{3}\mu_0 I}{\pi a}$

12. The maximum relativistic kinetic energy of $\beta$ particles from a radioactive nucleus is equal to the rest mass energy of the particle. A magnetic field is applied perpendicular to the beam of $\beta$ particles, which bends it to a circle of radius $R$. The field is given by :

(a) $3m_0c/eR$ (b) $\sqrt{2}m_0c/eR$ (c) $\sqrt{3}m_0c/eR$ (d) $\sqrt{3}m_0c/2eR$

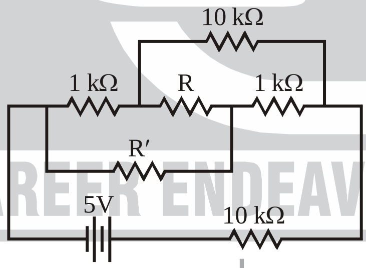
13. Light takes approximately 8 minutes to travel from the Sun to the Earth. Suppose in the frame of the Sun an event occurs at $t = 0$ at the Sun and another event occurs on Earth at $t = 1$ minute. The velocity of the inertial frame in which both these events are simultaneous is:
(a) $c/8$ with the velocity vector pointing from Earth to Sun
(b) $c/8$ with the velocity vector pointing from Sun to Earth
(c) The events can never be simultaneous - no such frame exists
(d) $c\sqrt{1-\left(\frac{1}{8}\right)^2}$ with velocity vector pointing from Sun to Earth

14. The central force which results in the orbit $r = a(1+\cos\theta)$ for a particle is proportional to :
(a) $r$ (b) $r^2$ (c) $r^{-2}$ (d) none of these

15. A spin-$\frac{1}{2}$ particle in a uniform external magnetic field has energy eigenstates $|1\rangle$ and $|2\rangle$. The system is prepared in ket-state $(|1\rangle+|2\rangle)/\sqrt{2}$ at time $t = 0$. It evolves to the state described by the ket $(|1\rangle-|2\rangle)/\sqrt{2}$ in time $T$. The minimum energy difference between two levels is:

(a) $\frac{h}{6T}$ (b) $\frac{h}{4T}$ (c) $\frac{h}{2T}$ (d) $\frac{h}{T}$

16. It is found that when the resistance $R$ indicated in the figure below is changed from 1 kΩ to 10 kΩ, the current flowing through the resistance $R'$ does not change. What is the value of the resistor $R'$ ?




(a) 5 kΩ (b) 100 kΩ (c) 10 kΩ (d) 1 kΩ



17. The sum of the infinite series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$ is

(a) $2\pi$ (b) $\pi$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

18. A spherical shell of radius $R$ carries a constant surface charge density $\sigma$ and is rotating about one of its diameters with an angular velocity $\omega$. The magnitude of the magnetic moment of the shell is:

(a) $4\pi\sigma\omega R^4$ (b) $4\pi\sigma\omega R^4/3$ (c) $4\pi\sigma\omega R^4/15$ (d) $4\pi\sigma\omega R^4/9$

19. Given a matrix $M = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, which of the following represents $\cos(\pi M/6)$?

(a) $\frac{1}{2}\begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix}$ (b) $\frac{\sqrt{3}}{4}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$ (c) $\frac{\sqrt{3}}{4}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$ (d) $\frac{1}{2}\begin{pmatrix}1 & \sqrt{3}\\ \sqrt{3} & 1\end{pmatrix}$

20. Consider $N$ non-interacting electrons $(N \sim N_A)$ in a box of sides $L_x, L_y, L_z$. Assume that the dispersion relation is $\varepsilon(k) = C k^4$, where $C$ is a constant, the ratio of the ground state energy per particle to the Fermi energy is :

(a) $\frac{3}{7}$ (b) $\frac{7}{3}$ (c) $\frac{3}{5}$ (d) $\frac{5}{7}$

21. A transistor in common base configuration has ratio of collector current to emitter current $\beta$ and ratio of collector to base current $\alpha$. Which of the following is true ?

(a) $\beta = \frac{\alpha}{(\alpha+1)}$ (b) $\beta = \frac{(\alpha+1)}{\alpha}$ (c) $\beta = \frac{\alpha}{(\alpha-1)}$ (d) $\beta = \frac{(\alpha-1)}{\alpha}$

22. You receive on average 5 emails per day during a 365 days year. The number of days on average on which you do not receive any emails in that year are:

(a) more than 5 (b) more than 2 (c) 1 (d) none of these

23. If $Y_{xy} = \frac{1}{\sqrt{2}}(Y_{2,2} - Y_{2,-2})$, where $Y_{l,m}$ are spherical harmonics, then which of the following is true ?

(a) $Y_{xy}$ is an eigenfunction of both $L^2$ and $L_z$ (b) $Y_{xy}$ is an eigenfunction of $L^2$ but not $L_z$

(c) $Y_{xy}$ is an eigenfunction of $L_z$ but not $L^2$ (d) $Y_{xy}$ is not an eigenfunction of either $L^2$ or $L_z$

24. A two dimensional box in a uniform magnetic field $B$ contains $N/2$ localised spin-$\frac{1}{2}$ particles with magnetic moment $\mu$, and $N/2$ free spinless particles which do not interact with each other. The average energy of the system at a temperature $T$ is :

(a) $3NkT - \frac{1}{2}N\mu B \sinh(\mu B/k_B T)$ (b) $NkT - \frac{1}{2}N\mu B \tanh(\mu B/k_B T)$

(c) $\frac{1}{2}NkT - \frac{1}{2}N\mu B \tanh(\mu B/k_B T)$ (d) $\frac{3}{2}NkT + \frac{1}{2}N\mu B \cosh(\mu B/k_B T)$



25. The value of the integral $\int_0^\infty \frac{\ell n\, x}{(x^2+1)}dx$ is :



(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{2}$ (c) $\pi^2$ (d) 0

**Part-B: 1-Mark Questions**

26. An ideal gas has a specific heat ratio $C_P/C_V = 2$. Starting at a temperature $T_1$ the gas under goes an isothermal compression to increase its density by a factor of two. After this an adiabatic compression increases its pressure by a factor of two. The temperature of the gas at the end of the second process would be:

(a) $\frac{T_1}{2}$ (b) $\sqrt{2}T_1$ (c) $2T_1$ (d) $\frac{T_1}{\sqrt{2}}$

27. The electric field $\vec{E} = E_0 \sin(\omega t - kz)\,\hat{x} + 2E_0 \sin(\omega t - kz + \pi/2)\,\hat{y}$ represents:

(a) a linearly polarized wave (b) a right hand circularly polarized wave
(c) a left hand circularly polarized wave (d) an elliptically polarized wave

28. If $\vec{k}$ is the wavefactor of incident light ($|\vec{k}| = 2\pi/\lambda$, $\lambda$ is the wavelength of light) and $\vec{G}$ is a reciprocal lattice vector, then the Bragg's law can be written as:
(a) $\vec{k} + \vec{G} = 0$ (b) $2\vec{k}\cdot\vec{G} + G^2 = 0$ (c) $2\vec{k}\cdot\vec{G} + k^2 = 0$ (d) $\vec{k}\cdot\vec{G} = 0$

29. The number of different Bravais lattices possible in two dimension is:
(a) 2 (B) 3 (c) 5 (d) 6

30. An electron confined within a thin layer of semiconductor may be treated as a free particle inside an infinitely deep one-dimensional potential well. If the difference in energies between the first and the second energy levels is $\delta E$, then the thickness of the layer is:
(a) $\sqrt{\frac{3\hbar^2\pi^2}{2m\delta E}}$ (b) $\sqrt{\frac{2\hbar^2\pi^2}{3m\delta E}}$ (c) $\sqrt{\frac{\hbar^2\pi^2}{2m\delta E}}$ (d) $\sqrt{\frac{\hbar^2\pi^2}{m\delta E}}$

31. The adjoint of a differential operator $\frac{d}{dx}$ acting on a wavefunction $\psi(x)$ for a quantum mechanical system is:
(a) $\frac{d}{dx}$ (b) $-i\hbar\frac{d}{dx}$ (c) $-\frac{d}{dx}$ (d) $i\hbar\frac{d}{dx}$

32. In the ground state of hydrogen atom, the most probable distance of the electron from the nucleus, in units of Bohr radius $a_0$ is:
(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) $\frac{3}{2}$

33. Circular fringes are obtained with a Michelson interferometer using 600nm laser light. What minimum displacement of one mirror will make the central fringe from bright to dark ?
(a) 600 nm (b) 300 nm (c) 150 nm (d) 120 Å

34. Given the condition $\nabla^2\Phi = 0$, the solution of the equation $\nabla^2\Psi = k\vec{\nabla}\Phi\cdot\vec{\nabla}\Phi$ is given by:
(a) $\Psi = k\Phi^2/2$ (b) $\Psi = k\Phi^2$ (c) $\Psi = k\Phi\,\ell n\,\Phi$ (d) $\Psi = k\Phi\,\ell n\,\Phi/2$

35. The output intensity $I$ of radiation from a single mode of resonant cavity obeys

$$\frac{d}{dt}I = -\frac{\omega_0}{Q}I,$$

where $Q$ is the quality factor of the cavity and $\omega_0$ is the resonant frequency. The form of the frequency spectrum of the output is:
(a) Delta function (b) Gaussian (c) Lorentzian (d) Exponential

36. For operators $P$ and $Q$, the commutator $[P, Q^{-1}]$ is:
(a) $Q^{-1}[P,Q]Q^{-1}$ (b) $-Q^{-1}[P,Q]Q^{-1}$ (c) $Q^{-1}[P,Q]Q$ (d) $-Q^{-1}[P,Q]Q^{-1}$

37. For a quantum mechanical harmonic oscillator with energies, $E_n = (n+1/2)\hbar\omega$, where $n = 0, 1, 2, ....,$ the partition function is:
(a) $\frac{e^{\hbar\omega/k_BT}}{e^{\hbar\omega/k_BT}-1}$ (b) $e^{\hbar\omega/2k_BT}-1$ (c) $e^{\hbar\omega/2k_BT}+1$ (d) $\frac{e^{\hbar\omega/2k_BT}}{e^{\hbar\omega/k_BT}-1}$

38. A semicircular piece of paper is folded to make a cone with the center of the semicircle as the apex. The half angle of the resulting cone would be:

(a) 90° (b) 60° (c) 45° (d) 30°

39. A spin-$\frac{1}{2}$ particle is in a state $\left(\left|\uparrow\right\rangle + \left|\downarrow\right\rangle\right)/\sqrt{2}$, where $\left|\uparrow\right\rangle$ and $\left|\downarrow\right\rangle$ are the eigenstates of $S_z$ operator. The exceptation value of the spin angular momentum measured along *x*-direction is:

(a) $\hbar$ (b) $-\hbar$ (c) 0 (d) $\frac{\hbar}{2}$

40. The half-life of a radioactive nuclear source is 9 days. The fraction of nuclei which are left undecayed after 3 days is:

(a) $\frac{7}{8}$ (b) $\frac{1}{3}$ (c) $\frac{5}{6}$ (d) $\frac{1}{2^{1/3}}$

41. If the Rydberg constant of an atom of finite nuclear mass is $\alpha R_\infty$, where $R_\infty$ is the Rydberg constant corresponding to an infinite nuclear mass, the ratio of the electronic to nuclear mass of the atom is:

(a) $\frac{(1-\alpha)}{\alpha}$ (b) $\frac{(\alpha-1)}{\alpha}$ (c) $(1-\alpha)$ (d) $\frac{1}{\alpha}$

42. A gas contains particles of type A with fraction 0.8, and particles of type B with fraction 0.2. The probability that among 3 randomly chosen particles at least one is of type A is:

(a) 0.8 (b) 0.25 (c) 0.33 (d) 0.992

43. A cylindrical shell of mass *m* has an outer radius *b* and an inner radius *a*. The moment of inertia of the shell about the axis of the cylinder is:

(a) $\frac{1}{2}m(b^2-a^2)$ (b) $\frac{1}{2}m(b^2+a^2)$ (c) $m(b^2+a^2)$ (d) $m(b^2-a^2)$

44. If the direction with respect to a right-handed cartesian coordinate system of the ket vector $\left|z,+\right\rangle$ is (0, 0, 1), then the direction of the ket vector obtained by application of rotations:

$\exp(-i\sigma_z\pi/2)\exp(i\sigma_y\pi/4)$, on the ket $\left|z,+\right\rangle$ is ($\sigma_y, \sigma_z$ are the Pauli matrices):

(a) $(0,1,0)$ (b) $(1,0,0)$ (c) $(1,1,0)/\sqrt{2}$ (d) $(1,1,1)/\sqrt{3}$

45. Suppose *yz*-plane forms the boundary between two linear dielectric media *I* and *II* with dielectric constant $\varepsilon_I = 3$ and $\varepsilon_{II} = 4$, respectively. If the electric field in region. *I* at the interface is given by $\vec{E}_1 = 4\hat{x} + 3\hat{y} + 5\hat{z}$, then the electric field $\vec{E}_{II}$ at the interface in region *II* is:

(a) $4\hat{x} + 3\hat{y} + 5\hat{z}$ (b) $4\hat{x} + 0.75\hat{y} - 1.25\hat{z}$

(c) $-3\hat{x} + 3\hat{y} + 5\hat{z}$ (d) $3\hat{x} + 3\hat{y} + 5\hat{z}$

46. How much force does light from a 1.8 W laser exert when it is totally absorbed by an object?

(a) $6.0\times10^{-9}\,N$ (b) $0.6\times10^{-9}\,N$ (c) $6.0\times10^{-8}\,N$ (d) $4.8\times10^{-9}\,N$

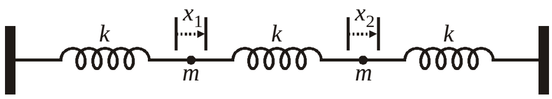
47. Self inductance per unit length of a long solenoid of radius *R* with *n* turns per unit length is:

(a) $\mu_0\pi R^2n^2$ (b) $2\mu_0\pi R^2n$ (c) $2\mu_0\pi R^2n^2$ (d) $\mu_0\pi R^2n$

48. In Millikan's oil-drop experiment an oil drop of radius *r*, mass *m* and charge $q = 6\pi\eta r(v_1+v_2)/E$ is moving upwards with a terminal velocity $v_2$ due to an applied electric field of magnitude *E*, where $\eta$ is the coefficients of viscosity. The acceleration due to gravity is given by :

(a) $g = 6\pi\eta r v_1/m$ (b) $g = 3\pi\eta r v_1/m$ (c) $g = 6\pi\eta r v_2/m$ (d) $g = 3\pi\eta r v_2/m$

49. For the coupled system shown in the figure, the normal coordinates are $x_1 + x_2$ and $x_1 - x_2$, corresponding to the normal frequencies $\omega_0$ and $\sqrt{3}\omega_0$, respectively.




At $t = 0$, the displacements are $x_1 = A, x_2 = 0$, and the velocities are $v_1 = v_2 = 0$. The displacement of the second particle at time $t$ is given by:

(a) $x_2(t) = \frac{A}{2}\left(\cos(\omega_0 t) + \cos\left(\sqrt{3}\omega_0 t\right)\right)$ (b) $x_2(t) = \frac{A}{2}\left(\cos(\omega_0 t) - \cos\left(\sqrt{3}\omega_0 t\right)\right)$

(c) $x_2(t) = \frac{A}{2}\left(\sin(\omega_0 t) - \sin\left(\sqrt{3}\omega_0 t\right)\right)$ (d) $x_2(t) = \frac{A}{2}\left(\sin(\omega_0 t) - \frac{1}{\sqrt{3}}\sin\left(\sqrt{3}\omega_0 t\right)\right)$

50. The mean value of random variable $x$ with probability density $p(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-(x^2 + \mu x)/(2\sigma^2)\right]$, is:

(a) 0 (b) $\frac{\mu}{2}$ (c) $-\frac{\mu}{2}$ (d) $\sigma$