IIT-JAM MATHEMATICS

Test : Differential Calculus

Time : 60 Minutes **Date : 06-08-2017**
M.M. : 40

INSTRUCTION:

1. Attempt all the questions.
2. **Section-A** contains **5 Multiple Choice Questions (MCQ)**. Each question has 4 choices (a), (b), (c) and (d), for its answer, out of which ONLY ONE is correct. From **Q.1 to Q.5** carries **1 Mark** each. For each incorrect answer **1/4th mark** will be deducted.
3. **Section-B** contains **5 Multiple Select Questions (MSQ)**. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE or MORE than ONE is/are correct. **Q.6 to Q. 10** for each correct answer you will be awarded **3 marks**. There is no negative marking in this section.
4. **Section-C** contains **5 Numerical Answer Type (NAT)** questions. **Q.11 to Q.15** carries **2 Marks** each. There is no negative marking in this section.
5. **Section-D** contains **5 True & False Questions**. **Q.16 to Q.20** carries **2 Marks** each. For each incorrect answer **–1 mark** will be deducted.

SECTION-A [Multiple Choice Questions]

1. Let $f:\mathbb{R}\to\mathbb{R}$ defined as

$$f(x)=\begin{cases} |x^2-2x| & ; \quad x\in Q^c \\ x & ; \quad x\in Q \end{cases}$$

Then f is continuous at

(a) $x=0,\ x=1,\ x=3$ (b) $x=0,\ x=1$ (c) $x=0,\ x=3$ (d) $x=3,\ x=1$

2. Let f be a function that is continuous everywhere and let

$$F(x)=\begin{cases} \dfrac{f(x)\sin^2 x}{x} & \text{if} \quad x\neq 0 \\ 0 & \text{if} \quad x=0 \end{cases}$$

Then, $F'(0)=$

(a) 0 (b) $f(0)$ (c) $f'(0)$ (d) does not exist

3. The domain of the function $f(x)=\dfrac{\ln(\ln(\ln x))}{x-3}+\sin x$ is

(a) $(0,3)$ (b) $(e,3)$ (c) $(0,3)\cup(3,\infty)$ (d) $(e,3)\cup(3,\infty)$

4. Suppose $\lim_{x\to a} f(x)$ and $\lim_{x\to a} f(x)\, g(x)$ are exist then $\lim_{x\to a} g(x)$ is

(a) exists (b) does not exist (c) always exist (d) none of these

5. Define $f:\mathbb{R}\to\mathbb{R}$ as follows

$$f(x)=\begin{cases}1 & \text{if} \quad x\in Q\\ \frac{\sin x}{x} & \text{if} \quad x\in Q^c\end{cases}$$

Then,

(a) f is continuous everywhere (b) f is continuous only at $x=0$

(c) f is continuous all rational points (d) f is continuous at all irrational points

SECTION-B [Multiple Select Questions]

6. If $y=a\log x+bx^2+x$ has its extremum value at $x=-1$ and $x=2$, then

(a) $a=2,\ b=-1$ (b) $a=2,\ b=-1/2$ (c) $a=-1/2,\ b=1/2$ (d) none of these

7. Consider $f(x)=|x|^3$, then which of the following is TRUE ?

(a) $f(x)$ is continuous but not differentiable at 0

(b) $f(x)$ is differentiable at 0 and $f'(0)=0$

(c) $f'(x)$ is also differentiable at $x=0$

(d) $f''(x)$ is also differentiable at $x=0$

8. Let $A=\{x\in\mathbb{R}: x>0\}$

$$h(x)=\begin{cases}0 & \text{if} \quad x\in A\cap Q^c\\ \frac{1}{n} & \text{where} \quad x=\frac{m}{n}\ (m \text{ and } n \text{ have no common factor other than 1) and } x\in A\cap Q\end{cases}$$

Then,

(a) h is continuous everywhere

(b) h is discontinuous everywhere

(c) h is continuous for rationals in A and discontinuous for irrationals in A

(d) h is continuous for irrationals in A and discontinuous for rationals in A

9. Consider the statement :

S_1 : Let $f(x)=x$ and $g(x)=\sin x$, then both f and g are uniformly continuous on $\mathbb{R}$

S_2 : Define $h(x)=x\sin x$, then $h(x)$ is also uniformly continuous on $\mathbb{R}$

(a) Only S_1 is true (b) Only S_2 is true (c) Both are true (d) Both are false

10. Which of the following maps are differentiable everywhere ?

(a) $f(x)=|x|^3 x,\ x\in R$ (b) $f:R\to R$ such that $|f(x)-f(y)|\le|x-y|^{\sqrt{2}}\ \forall\, x\in\mathbb{R}$

(c) $f(x)=x^3\sin\frac{1}{\sqrt{|x|}}$; $x\neq 0$ and $f(0)=0$ (d) none of these

SECTION-C [Numerical Answer Type]

11. The value of $\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\sin^2 x}\right)$ is ___________

12. A function $f:\mathbb{R}\to\mathbb{R}$ is defined by $f(x)=|\sin x|+|\cos x|\ \forall\ x\in\mathbb{R}$, then the point(s) where f is not differentiable is/are ____________

13. Let $f:\mathbb{R}\to\mathbb{R}$ be such that $f(x)=x+x^2+|x-1|\ \forall x\in\mathbb{R}$, then f is not differentiable at x = _____

14. Let $f:(0,\infty)\to\mathbb{R}$ be continuous function such that $\int_0^x f(t)dt=-2+\frac{x^2}{2}+4x\sin 2x+3\cos 2x$, then the value of $\frac{1}{(\pi-8)}f\left(\frac{\pi}{4}\right)$ is __________

15. If f be a real valued differentiable function on $[a,\infty)$ where $a\geq 1$ such that $f(1)=3$, if $2\int_2^x f(t)dt=xf(x)+x^3\ \forall x\geq 1$, then $f(2)=$ ______________

SECTION-D [True & False]

16. The function $f(x)=|x|^{1/2}x$ is differentiable at $x=0$

17. Let $f(x)=x\sin\left(\frac{1}{x}\right):\forall x\in(0,1]$, then f is not uniformly continuous on $(0,1]$.

18. Let $I\subseteq\mathbb{R}$ be an interval and a function $f:I\to\mathbb{R}$ is differentiable on I such that f' is monotonic on I, then f' is continuous on I.

19. Let $f:[a,b]\to[a,b]$ be differentiable and assume that $f'(x)\neq 1$ for $x\in(a,b)$, then f has a unique fixed point in $[a,b]$.

20. If $f:\mathbb{R}\to\mathbb{R}$ is differentiable and bijective, then f^{-1} is also differentiable.

IIT-JAM MATHEMATICS
Test : Differential Calculus

Date : 06-08-2017

ANSWER KEY

SECTION-A [Multiple Choice Questions]

1. (a) **2.** (b) **3.** (d) **4.** (b) **5.** (b)

SECTION-B [Multiple Select Questions]

6. (b) **7.** (b, c) **8.** (d) **9.** (a)
10. (a, b, c)

SECTION-C [Numerical Answer Type]

11. (–0.33) **12.** $x = n\pi, n\pi + \frac{\pi}{2},$ where $n \in \mathbb{Z}$
13. (1) **14.** (0.25) **15.** (0)

SECTION-D [True & False]

16. (True) **17.** (False) **18.** (True) **19.** (True)
20. (False)