IIT-JAM MATHEMATICS

Test : Differential Equation

Time : 60 Minutes **Date : 08-06-2017**

M.M. : 50

INSTRUCTION:

1. Attempt all the questions.
2. **Section-A** contains **15 Multiple Choice Questions (MCQ)**. Each question has 4 choices (a), (b), (c) and (d), for its answer, out of which ONLY ONE is correct. From **Q.1 to Q.5** carries **1 Mark** and **Q.6 to Q.15** carries **2 Marks** each. For each incorrect answered **1/4th mark** will be deducted.
3. **Section-B** contains **5 Multiple Select Questions (MSQ)**. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE or MORE than ONE is/are correct. **Q.16 to Q. 20** for each correct answer you will be awarded **3 marks**. There is no negative marking in this section.
4. **Section-C** contains **5 Numerical Answer Type (NAT)** questions. **Q.21 to Q.25** carries **2 Marks** each. There is no negative marking in this section.

SECTION-A [Multiple Choice Questions]

1. Let $y = e^{(y'+y'')}$, then sum of degree and order of given ordinary differential equation is

   (a) 3 (b) 2 (c) 1 (d) can not determine

2. The differential equation of the system of parabolas $y^2 = 4a(x-b)$ is given by

   (a) $\frac{dy}{dx} = \frac{2a}{y}$ (b) $y^2\frac{d^2y}{dx^2} + 2a\frac{dy}{dx} = 0$ (c) $yy'' + (y')^2 = 0$ (d) $(y')^2 + y'' = 0$

3. The differential equation $\left(\frac{1}{x^2} + \frac{1}{y^2}\right)dx + \left(\frac{Ax+1}{y^3}\right)dy = 0$ is exact and has the solution.

   (a) $A = -2$ and $2x^2 - 2y^2 - x = cxy^2$, c is constant

   (b) $A = +2$ and $2x^2 - 2y^2 - x = cxy$, c is constant

   (c) $A = -2$ and $2x^2 + 2y^2 - x = cxy$, c is constant

   (d) $A = 2$ and $2x^2 + 2y^2 - x = cxy^2$, c is constant

4. Let the general solution of a differential equation be $y = ae^{bx+c}$, then order of the differential equation is

   (a) 1 (b) 2 (c) 3 (d) can not say

5. Let $y' = ye^x$ be the differential equation, let y be a solution passing through $(0, e)$, then $y(1)$ is

   (a) e (b) e^e (c) 1 (d) 0

6. The order of the differential equation of all circles of given radius a is

(a) 1 (b) 2 (c) 3 (d) 4

7. The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents a family of

(a) straight line (b) circles (c) parabolas (d) ellipses

8. The differential equation $xdy - ydx = \sqrt{x^2 + y^2}\,dx$ is

(a) Homogeneous equation (b) Variable separable equation

(c) Exact differential equation (d) Linear equation

From Q. No. 9 to Q. No. 13, each question contains, Statement-I (Assertion) and Statement-II (Reason). Each question has 4 choices (a), (b), (c) and (d), out of which only one is correct. So select the correct choices.

Choices are

(a) If Statement-I is true, Statement-II is correct explanation of Statement-I.

(b) If Statement-I is true, but Statement-II is not correct explanation of Statement-I.

(c) If Statement-I is true, Statement-II is false

(d) If Statement-I is false, Statement-II is true

9. Statement-I : The order of the differential equation formed by the family of curve $y = c_1e^x + (c_2 + c_3)e^{x+c_4}$ is 1. Here c_1, c_2, c_3, c_4 are arbitrary constant.

Statement-II : The order of differential equation formed by any family of curve is equal to the number of arbitrary constants present in it.

10. Statement-I : The degree of differential equation $3\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \log\left(\frac{d^2y}{dx^2}\right)$ is not defined.

Statement-II : The degree of differential equation is the power of highest order derivative when differential equation has expressed as polynomial of derivatives.

11. Statement-I : The differential equation $y^3dy + (x + y^2)dx = 0$ becomes homogeneous if we put $y^2 = t$.

Statement-II : All differential equation of first order and first degree becomes homogenous if we put $y = tx$.

12. Statement-I : The differential equation of the family of curves represented by $y = Ae^x$ is given by $\frac{dy}{dx} = y$.

Statement-II : $\frac{dy}{dx} = y$ is valid for every member of the given family.

13. Statement-I : Solution of differential equation $dy(x^2y - 1) + dx(y^2x - 1) = 0$ is $\frac{x^2y^2}{2} = x + y + c$.

Statement-II : Order of differential equation of family of circle touching the coordinate axis is 1.

14. The solution of $\frac{xdx-ydy}{xdy-ydx}=\sqrt{\frac{1+x^2-y^2}{x^2-y^2}}$ is

(a) $\sqrt{x^2-y^2}+\sqrt{1+x^2-y^2}=\frac{c(x+y)}{\sqrt{x^2-y^2}}$

(b) $\sqrt{x^2-y^2}+\sqrt{1-x^2+y^2}=\frac{c(x+y)}{\sqrt{x^2-y^2}}$

(c) $\sqrt{x^2-y^2}-\sqrt{1+x^2-y^2}=\frac{c(x-y)}{\sqrt{x^2-y^2}}$

(d) $\sqrt{x^2-y^2}-\sqrt{1-x^2+y^2}=\frac{c(x-y)}{\sqrt{x^2-y^2}}$

15. The solution of $\frac{dy}{dx}+\sin\left(\frac{x+y}{2}\right)=\sin\left(\frac{x-y}{2}\right)$ is

(a) $\ln\left|\tan\frac{y}{4}\right|=c+2\sin\frac{x}{2}$

(b) $\ln\left|\tan\frac{y}{4}\right|=c-2\sin\frac{x}{2}$

(c) $\ln\left|\tan\frac{y}{4}\right|=c+2\cos\frac{x}{2}$

(d) $\ln\left|\tan\frac{y}{4}\right|=c-2\cos\frac{x}{2}$

SECTION-B [Multiple Select Questions]

16. Consider the family of all circles whose centres lie on the straight line $y=x$, if this family of circles is represented by the differential equation $Py''+Qy'+1=0$, where P, Q are functions of x, y and y', then which of the following statements is / are true?

(a) $P=y+x$

(b) $P=y-x$

(c) $P+Q=1-x+y+y'+(y')^2$

(d) $P-Q=x+y-y'-(y')^2$

17. A tangent drawn to the curve, $y=f(x)$ at $P(x,y)$ cuts the x-axis and y-axis at A and B respectively such that $BP:AP=3:1$, given that $f(1)=1$, then

(a) Equation of the curve is $x\frac{dy}{dx}-3y=0$

(b) Equation of the curve is $x\frac{dy}{dx}+3y=0$

(c) Curve passes through (2, 1/8)

(d) Normal at (1, 1) is $x+3y=y$

18. The solution of primitive integral equation $(x^2+y^2)dy=xydx$ is $y=y(x)$. If $y(1)=1$ and $y(x_0)=e$ then x_0 is

(a) $\sqrt{2(e^2-1)}$

(b) $\sqrt{2(e^2+1)}$

(c) $\sqrt{3}e$

(d) $\sqrt{\frac{e^2+1}{2}}$

19. The initial value problem $y'=\sqrt{y}, y(0)=\alpha, \alpha>0$ has

(a) at least two solutions if $\alpha=0$

(b) no solution if $\alpha>0$

(c) at least one solution if $\alpha>0$

(d) a unique solution if $\alpha=0$

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20. A solution of $(x^2y^2 + y^4 + 2x)dx + 2y(x^3 + xy^2 + 1)dy = 0$ is

(a) $x^2 + \log|x^2 - y^2| = \text{constant}$ (b) $x^2y + \log|x^2 - y^2| = \text{constant}$

(c) $x^2y + \log(x^2 + y^2) = \text{constant}$ (d) $xy^2 + \log(x^2 + y^2) = \text{constant}$

SECTION-C [Numerical Answer Type]

21. The order and degree of $\sin\left(\frac{dy}{dx} + \frac{d^2y}{dx^2}\right) = y$ is _____

22. The integrating factor of $y(xy + 2(xy)^2)dx + x(xy - (xy)^2)dy = 0$ is $(xy)^\alpha$, then α is _____

23. The solution of the differential equation $\frac{dy}{dx} = e^{x+y}, y(1) = 1$ at $x = -1$ is _____

24. Let $y' = (x + y)^2$ and $y(0) = 0$, then $y\left(\frac{\pi}{4}\right) + \frac{\pi}{4}$ is _____

25. The degree of the differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where c is a positive parameter is _____

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ANSWER KEY

SECTION-A [Multiple Choice Questions]

1. (a)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (b)
6. (b)	**7.** (c)	**8.** (a)	**9.** (c)	**10.** (a)
11. (c)	**12.** (a)	**13.** (b)	**14.** (a)	**15.** (b)

SECTION-B [Multiple Select Questions]

16. (b, c)	**17.** (b, c)	**18.** (c)	**19.** (a, c)	**20.** (d)

SECTION-C [Numerical Answer Type]

21. (Order 2, Degree 1)	**22.** $(\alpha = -3)$	**23.** (–1)	**24.** (+1)	**25.** (3)

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