Target IIT-JAM-2018

Test Series-1

Booklet Code: A

Real Analysis + Modern Algebra

Duration: 3:00 Hours **MATHEMATICS-MA** **Date: 04-01-2018**

Maximum Marks: 100

Read the following instructions carefully:

1 Attempt all the questions.

2. **Section-A** contains **30** Multiple Choice Questions (MCQ). Each question has 4 choices (a), (b), (c) and (d), for its answer, out of which **ONLY ONE** is correct. From **Q.1 to Q.10** carries 1 Marks and **Q.11 to Q.3**0 carries 2 Marks each.

3. **Section-B** contains **10** Multiple Select Questions(MSQ). Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which **ONE or MORE than ONE** is/are correct. For each correct answer you will be awarded **2 marks.**

4. Section-**C** contains **20** Numerical Answer Type (NAT) questions. From **Q.41 to Q.50** carries **1 Mark** each and **Q.51 to Q.60** carries **2 Marks** each. For each NAT type question, the value of answer in between 0 to 9.

5. In all sections, questions not attempted will result in zero mark. In Section–A (MCQ), wrong answer will result in negative marks. For all **1 mark** questions, **1/3 marks** will be deducted for each wrong answer. For all **2 marks** questions, **2/3 marks** will be deducted for each wrong answer. In Section–B (MSQ),there is no negative and no partial marking provision. There is no negative marking in Section –C (NAT) as well.

Regn. No. | | | | | | | | | | |

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SECTION-A

[Multiple Choice Questions (MCQ)]

Q. 1 – Q. 10 carry one mark each.

1. Which of the following is not a subgroup of $(\mathbb{C}, +)$

(a) $(\mathbb{R}, +)$
(b) $(G, +)$, where $G = \{\pi \cdot r \mid r \in \mathbb{Q}\}$
(c) $(G, +)$, where $G = \{i\, r \mid r \in \mathbb{R}\}$
(d) $(G, +)$, where $G = \{\pi^n \mid n \in \mathbb{Z}\}$

2. Let $a_n = (-1)^n \cdot \sin\left(\frac{n\pi}{2}\right); n = 1, 2, \ldots$ then

(a) $\liminf a_n = -1$ and $\limsup a_n = 1$
(b) $\liminf a_n = 0$ and $\limsup a_n = 1$
(c) $\liminf a_n = -1$ and $\limsup a_n = 0$
(d) $\liminf a_n = 1$ and $\limsup a_n = 0$

3. Let $a_n = \frac{4^{3n}}{3^{4n}}; n \in \mathbb{N}$, then the sequence $\langle a_n \rangle$

(a) is unbounded
(b) is bounded but not convergent
(c) converges to 0
(d) converges to 1

4. How many elements does the set $\{Z \in C \mid Z^{60} = 1, Z^k \neq 1 \text{ for } 0 < k < 60\}$ have

(a) 24 (b) 30 (c) 32 (d) 16

5. Infinite series $\sum \frac{\lfloor n}{n^n} x^n$ converges if

(a) $|x| < 1$ (b) $|x| < \ln 2$ (c) $|x| < 2$ (d) None

6. Let G be a group of order 9 and $f(x) = x^2 \forall x \in G$ then f is

(a) Not one-one
(b) Not onto
(c) Not a homomorphism
(d) One-one, onto and homomorphism

7. Consider the following subsets of $\mathbb{R}$

$E = \left\{\frac{n}{n+1}; n \in \mathbb{N}\right\} \cup \{1\}$ and $F = \left\{\frac{1}{1-x}; 0 \leq x < 1\right\}$. Then

(a) Both E and F are closed
(b) E is closed and F is not closed
(c) E is not closed but F is closed
(d) Neither E nor F is closed

8. If $x_n = 1 + (-1)^n + \frac{1}{3^n}$, then

(a) $\limsup (x_n) = 1$
(b) $\liminf (x_n) = -1$
(c) $\langle x_n \rangle$ is a convergent sequence
(d) $\limsup(x_n) \neq \liminf(x_n)$

9. The order of automorphism group $\text{Aut}(\mathbb{Z}_2 \otimes \mathbb{Z}_4)$ is

(a) 4 (b) 6 (c) 8 (d) 10

10. Let f be a homomorphism from $(\mathbb{Z},+)$ to $(\mathbb{Q}-\{0\},\times)$ such that $f(2)=\frac{1}{5}$, then the value of $f(-6)$ is

(a) 125 (b) 1/125 (c) 5 (d) 1/5

Q. 11 – Q. 30 carry two marks each.

11. Let G be finite group, an element $a\in G$ is called a square if $\exists\, x\in G$ s.t. $x^2=a$ then

(a) If $a,b\in G$ are not square, then $ab\in G$ is square
(b) If G is cyclic, if $a,b\in G$ are not square then $ab\in G$ is square
(c) G has a normal subgroup.
(d) If every proper subgroup of G is cyclic then G is cyclic.

12. $\sum_{n=1}^{\infty} P^n\cdot n^P\,(P>0)$ converges if

(a) $P<1$ (b) $P\le 1$ (c) $2>P>1$ (d) $P>2$

13. The series $x+\frac{2^2\cdot x^2}{\lfloor 2}+\frac{3^3\cdot x^3}{\lfloor 3}+\frac{4^4\cdot x^4}{\lfloor 4}+....$ is convergent if

(a) $0<x<\frac{1}{e}$ (b) $x>\frac{1}{e}$ (c) $\frac{2}{e}<x<\frac{3}{e}$ (d) $\frac{3}{2}<x<\frac{4}{e}$

14. Pick out the cases where the given subgroup H is normal subgroup of the group G.

(a) G is the group of all 2×2 invertible upper traingular matrices with real enteries, uder matrix multiplication and H is the subgroup of all such matrices $[a_{ij}]$ such that $a_{11}=1$
(b) G is the group of all 2×2 invertible upper triangular matrices with real entries, under matrix multiplication and H is the subgroup of all such matrices $[a_{ij}]$ such that $a_{11}=a_{22}$
(c) G is the group of all 2×2 invertible upper triangular matrices with real entries, under matrix multiplication and H is the subgroup of all such matrices $[a_{ij}]$ with positive determinant.
(d) all of the above.

15. Which has a subgroup isomorphism to S_5.

(a) A_7 (b) A_8 (c) A_9 (d) all of the above

16. Let G be a non-abelian group of order 39. Then which of the following statements about G is true.

(a) $Z(G)\neq\{e\}$
(b) G has a normal subgroup of order 13
(c) G does not have a normal subgroup of order 3
(d) G have three subgroup of order 3

17. Let $\langle x_n\rangle$ be the sequence defined by $x_1=2$, and $x_{n+1}=\frac{1}{2}\left(x_n+\frac{4}{x_n}\right)$. Then

(a) $\langle x_n\rangle$ converges to $2\sqrt{2}$ (b) $\langle x_n\rangle$ is an increasing sequence
(c) $\langle x_n\rangle$ is a decreasing sequence (d) $\langle x_n\rangle$ converges to 2

18. Let $P(x_1, x_2, x_3, x_4)$ be any polynomial with integer coefficient in the independent variable x_1, x_2, x_3, x_4. Each $\sigma \in S_4$ given a permutation of $\{x_1, x_2, x_3, x_4\}$ by defining $\sigma \cdot x_i = x_{\sigma(i)}$.

If $P(x_1, x_2, x_3, x_4) = 12x_1^5x_2^7x_4 - 18x_2^3x_3 + 11x_1^6x_2x_3^3x_4^{23}$ and $\sigma = (12)(34)$, then $\sigma \cdot P(x_1, x_2, x_3, x_4)$ is equal to

(a) $12x_1^7x_3^5x_2 - 18x_2^3x_3 + 11x_2^6x_3x_4^3x_2^{23}$ (b) $12x_1^5x_2^7x_4 - 18x_2^3x_3 + 11x_1^6x_2x_3^3x_4^{23}$

(c) $12x_1^7x_2^5x_3 - 18x_1^3x_4 + 11x_1x_2^6x_3^{23}x_4^3$ (d) None of these

19. Given S_1 and S_2 where

S_1: A series $\sum_{n=1}^{\infty} a_n$ converges if for a given $\in > 0$ there exists $N_0 \in \mathbb{N}$ such that $|a_{n+1} - a_n| < \in$ for all $n \geq N_0$.

S_2 : A series $\sum_{n=1}^{\infty} a_n$ converges if $|a_{n+1} - a_n| < \alpha^n$, where α is fixed real number in (0, 1).

Which of the following statements are true?

(a) S_1 is true but S_2 is false (b) S_1 is false but S_2 is true

(c) Both S_1 and S_2 are true (d) Both S_1 and S_2 are false

20. Let σ is the m-cycle $(a_1, a_2 ... a_m)$, then for all $i \in \{1, 2, 3 ... m\}$. Choose the correct statement

(a) $\sigma^i(a_K) = a_{K+1}$ (b) $\sigma^i(a_K) = a_{K+i}$

(c) $\sigma^i(a_K) = a_r$ where $K + i \equiv r (\bmod m)$ (d) None of these

21. The quotient group $\dfrac{\mathbb{Q}_8}{\{1, -1\}}$ is isomorphic to

(a) $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ (b) $\mathbb{Q}_8$

(c) $\mathbb{Z}_4$ (d) $\{1, -1\}$

22. Let $S \subseteq \mathbb{R}$ and ∂S denote the boundary of S and $\overline{S}$ denote the closure of S. Then which one of the following is false.

(a) $\partial Q = \mathbb{R}$ (b) $\partial(\mathbb{R} \setminus T) = \partial T,\ T < \mathbb{R}$

(c) $\partial(T \cup V) = \partial T U \partial V, T, V \leq \mathbb{R}$ (d) $\partial T = \overline{T} \cap (\overline{\mathbb{R} \setminus T}), T < \mathbb{R}$

23. Let G be the set of all 4 × 4 orthogonal matrices and let $H = \{A \in G; |A| = 1\}$.

Then which of the following is not true

(a) G is group under matrix multiplication

(b) H is a normal subgroup of G

(c) G/H is non-abelian

(d) $\phi : G \to \{-1, 1\}$ given by $\phi(A) = \det(A)$ is onto

24. Let S be the collection of all sequences whose terms are the integers 0 and 1. Then

(a) S is countable
(b) S is uncountable
(c) There is a onto map from $\mathbb{N}$ to S
(d) There is a one-one map from S to $\mathbb{N}$

25. If m and M are representing the infimum and suprimum of the set $S=\left\{\frac{2x+3}{x+2}; x\geq 0\right\}$, then

(a) $m\in S, M\notin S$
(b) $m\notin S, M\notin S$
(c) $m\notin S, M\in S$
(d) $m\in S, M\in S$

26. Let G be a finite group having a normal subgroup of order 2, then order of centre of G is

(a) 1
(b) an odd integer > 1
(c) An even integer ≥ 2
(d) can't say

27. For any group G, let Aut (G) denote the group of automorphism of G. Which of the following is true ?

(a) If Aut (G) is trivial, then G is trivial
(b) Aut ($\mathbb{Z}$) is isomorphic to $\mathbb{Z}$
(c) If G is cyclic then Aut (G) is cyclic
(d) Aut ($\mathbb{Z}$) is isomorphic to $\mathbb{Z}_2$

28. The set $\left\{x\in(-\pi,\pi); |\sin x| > \frac{1}{2}\right\}$ is

(a) An open interval
(b) A union of finitely many disjoint open intervals
(c) A union of contably infinite disjoint open intervals
(d) A union of uncontably infinite disjoint open intervals

29. For a real number x we let $[x]$ denote the largest integer not exceeding x. For a natural number n, let $a_n=\frac{[n\sqrt{2}]}{n}$.

The limit $\lim\limits_{n\to\infty} a_n$

(a) equals 0
(b) equals $[\sqrt{2}]$
(c) equals $\sqrt{2}$
(d) None

30. Using the fact that $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}=\log 2$, $\sum_{n=1}^{\infty}\frac{(-1)^n}{n(n+1)}$ equals

(a) $1-2\log 2$
(b) $1+\log 2$
(c) $(\log 2)^2$
(d) $-(\log 2)^2$

SECTION-B

[Multiple Select Questions (MSQ)]

Q. 31 – Q. 40 carry two marks each.

31. $\phi : G \to G'$ be a group homomorphism. Then

(a) N is normal in $G \Rightarrow \phi(N)$ is normal in $\phi(G)$

(b) N is normal in $G \Rightarrow \phi(N)$ is normal in G'

(c) N' is normal in $\phi(G) \Rightarrow \phi^{-1}(N')$ is normal in G

(d) N' is normal in $G' \Rightarrow \phi^{-1}(N')$ is normal in G

32. Consider the sequences $\langle a_n \rangle$ and $\langle b_n \rangle$, where

$a_n = \left(1+\frac{1}{n}\right)^n$ and $b_n = \left(1+\frac{1}{n}\right)^{n+1}$ $\forall n \in \mathbb{N}$. Then

(a) both sequence are monotonically increasing

(b) both sequence are monotonically decreasing

(c) $\langle a_n \rangle$ is monotonically increasing and $\langle b_n \rangle$ is monotonically decreasing

(d) $\langle b_n \rangle$ is monotonically increasing and $\langle a_n \rangle$ is monotonically decreasing

33. Consider the linear congruence equation $51x \equiv 12 \pmod{87}$ then

(a) it has 3 incongruent solution

(b) The set of incongruent solution is {19, 48, 77}

(c) This equation has the same solution as those of $17x \equiv 4 \pmod{29}$

(d) All of above

34. Which of the following statements is / are true?

(a) There exist a connected set in $\mathbb{R}$, which is not compact

(b) Every bounded infinite subset V of $\mathbb{R}$ has limit point in V itself

(c) There exist a compact set in $\mathbb{R}$ which is not connected

(d) Arbitrary union of closed intervals in $\mathbb{R}$ need not be compact

35. Let $\langle x_n \rangle$ and $\langle y_n \rangle$ are sequences of real numbers, which of the following is/are true?

(a) $\limsup(x_n + y_n) \le \limsup x_n + \limsup y_n$

(b) $\limsup(x_n + y_n) \ge \limsup x_n + \limsup y_n$

(c) $\liminf(x_n + y_n) \le \liminf x_n + \liminf y_n$

(d) $\liminf(x_n + y_n) \ge \liminf x_n + \liminf y_n$

36. Let G be a group of order 60 pick out the true statement

(a) G is abelian

(b) G has a subgroups of order 2, 3 and 5

(c) G has subgroup of order 30

(d) G has subgroups of order 6, 10 and 15

37. Which of the following are subgroups of $GL_3(\mathbb{C})$?

(a) $H = \{A \in M_3(\mathbb{C}) \mid \det(A) = 2^{\ell} \quad \ell \in \mathbb{Z}\}$ (b) $H = \left\{ \begin{bmatrix} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{bmatrix} \mid x, y, z \in \mathbb{C} \right\}$

(c) $H = \left\{ \begin{bmatrix} 1 & 0 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mid x \in \mathbb{C} \right\}$ (d) none of these

38. Let $A, B \subset [0,1]$ be two uncountable sets. Which of the following are false statements?

(a) If $A \cap B = \phi$, then either $\sup A \le \inf B$ or $\sup B \le \inf A$

(b) If $A \cap B = \phi$, then $[0,1] \setminus (A \cup B)$ is countable

(c) If $\inf A > \inf B$ and $\sup A = \sup B$, then $A \cap B \ne \phi$

(d) If $A \subset B$, then B/A is countable

39. Which of the following statements is/are correct?

(a) Let $b \in \mathbb{R}$ such that $0 < b < 1$, then $\lim\limits_{n \to \infty}(n \cdot b^n) = 0$

(b) If $\langle x_n \rangle$ is unbounded sequence, then always there exist a subsequence $\langle x_{n_k} \rangle$ such that $\lim \left(\frac{1}{x_{n_k}} \right) = 0$

(c) Let $\langle a_n \rangle$ be a sequence of real number such that $\lim\limits_{n \to \infty} \left(\frac{a_n}{n} \right) = \frac{1}{2}$, then $\langle a_n \rangle$ is convergent

(d) Suppose that $\sum a_n$ (where $a_n > 0 \forall n$) is convergent. then $\sum b_n$ where $b_n = \frac{a_1 + a_2 + ... + a_n}{n}$ is always divergent.

40. Let σ be the 12-cycles (1 2 3 4 5 6 7 8 9 10 11 12). For which positive integers i, σ^i is also a 12 cycle?

(a) 5 (b) 7 (c) 9 (d) 11

SECTION-C

[Numerical Answer Type (NAT)]

Q. 41 – Q. 50 carry one mark each.

41. $\limsup \left\{ \frac{(-1)^n}{2^n}; n \in \mathbb{N} \right\}$ is ________

42. If set A contains 6 elements and set B contains 4 element then number of onto function from A to B are ______

43. $\lim_{n\to\infty}\frac{\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\cdots\frac{n}{n+1}}{n}=$ __________

44. The sum of series $\frac{1}{30}+\frac{1}{42}+\frac{1}{56}+\frac{1}{72}+.....$ is __________

45. The last digit of 97^{2018} is __________

46. Consider the quotient group $\frac{\mathbb{Q}}{\mathbb{Z}}$, the additive group of rational number. Then the order of element $\frac{3}{5}+\mathbb{Z}$ in $\frac{\mathbb{Q}}{\mathbb{Z}}$ is __________

47. The limit of the sequence $\left(\frac{\lfloor n}{n^n\cdot e^{-n}}\right)^{1/n}$ is __________

48. The number of element of order 5 in non-abelian group of order 10 is __________

49. Number of cyclic sub-group of D_8 are __________

50. $\lim_{n\to\infty}\frac{(\lfloor n)^{1/n}}{n}=$ __________
(e = 2.7182)

Q. 51 – Q. 60 carry two marks each.

51. The sum of series $1+\frac{5}{3}+\frac{5}{3}\cdot\frac{7}{6}+\frac{5}{3}\cdot\frac{7}{6}\cdot\frac{9}{9}+$ ... is equal to __________
($\sqrt{7}$ = 2.646)

52. The sum of the series $\sum_{n=1}^{\infty}\left(-\log\left(1-\frac{1}{n^2}\right)\right)$ is __________
(log 2 = 0.3010)

53. The number of mutually non isomorphic abelian group of order 19^5 is __________

54. In A_4 number of elements satisfying the equation $x^4=e$ are __________

55. Consider the sequence $\langle x_n\rangle$ given by $4,\sqrt{4},\sqrt{4+\sqrt{4}},\sqrt{4+\sqrt{4+\sqrt{4}}},........$ then $\limsup\langle x_n\rangle+\liminf\langle x_n\rangle$ is __________

56. Let $\sigma=(1\,3\,5\,7\,11)\,(2\,4\,6)\in S_{11}$, then the smallest positive integer 'n' such that $\sigma^n=\sigma^{37}$ is __________

57. The value of $\lim_{n\to\infty}\frac{1\cdot 1!+2\cdot 2!+.......n\cdot n!}{(n+1)!}$ is __________

58. Number of group homomorphism from symmetric group S_3 to additive group $\frac{\mathbb{Z}}{6\mathbb{Z}}$ is __________

59. The sum of the squares of the roots of the qubic equation $x^3-4x^2+6x+1=0$ is __________

60. Let $\langle a_n\rangle$ be a sequence of positive real numbers such that $\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=r<1$. Then $\lim_{n\to\infty}a_n=$ __________

END OF THE QUESTION PAPER

SPACE FOR ROUGH WORK

IIT-JAM MATHEMATICS
TEST SERIES - 1
(Real Analysis + Modern Algebra)

Time : 3 Hours **Date : 04-01-2018**
M.M. : 100

ANSWER KEY

SECTION-A

[Multiple Choice Questions (MCQ)]

1. (d)	**2.** (a)	**3.** (c)	**4.** (d)	**5.** (c)
6. (d)	**7.** (a)	**8.** (d)	**9.** (b)	**10.** (a)
11. (b)	**12.** (a)	**13.** (a)	**14.** (d)	**15.** (d)
16. (b)	**17.** (d)	**18.** (c)	**19.** (b)	**20.** (c)
21. (a)	**22.** (c)	**23.** (c)	**24.** (b)	**25.** (a)
26. (c)	**27.** (d)	**28.** (b)	**29.** (c)	**30.** (a)

SECTION-B

[Multiple Select Questions (MSQ)]

31. (a, c)	**32.** (c)	**33.** (a, b, c, d)	**34.** (a, c, d)
35. (a, d)	**36.** (b, c)	**37.** (a, b, c)	**38.** (a, b, c, d)
39. (a, b, d)	**40.** (a, b, d)		

SECTION-C

[Numerical Answer Type (NAT)]

41. (0)	**42.** (1560)	**43.** (1)	**44.** (0.20)	**45.** (9)
46. (5)	**47.** (0)	**48.** (4)	**49.** (12)	**50.** (0.368)
51. (13.23)	**52.** (0.3010)	**53.** (7)	**54.** (4)	**55.** (1)
56. (7)	**57.** (1)	**58.** (2)	**59.** (16)	**60.** (0)