Target IIT-JAM-2018

Test Series-6

Full Length Test Series-3

Booklet Code: F

Duration: 3:00 Hours **MATHEMATICS-MA** **Date: 28-01-2018**

Maximum Marks: 100

Read the following instructions carefully:

1 Attempt all the questions.

2. **Section-A** contains **30** Multiple Choice Questions (MCQ). Each question has 4 choices (a), (b), (c) and (d), for its answer, out of which **ONLY ONE** is correct. From **Q.1 to Q.10** carries 1 Marks and **Q.11 to Q.30** carries 2 Marks each.

3. **Section-B** contains **10** Multiple Select Questions(MSQ). Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which **ONE or MORE than ONE** is/are correct. For each correct answer you will be awarded **2 marks.**

4. Section-**C** contains **20** Numerical Answer Type (NAT) questions. From **Q.41 to Q.50** carries **1 Mark** each and **Q.51 to Q.60** carries **2 Marks** each. For each NAT type question, the value of answer in between 0 to 9.

5. In all sections, questions not attempted will result in zero mark. In Section–A (MCQ), wrong answer will result in negative marks. For all **1 mark** questions, **1/3 marks** will be deducted for each wrong answer. For all **2 marks** questions, **2/3 marks** will be deducted for each wrong answer. In Section–B (MSQ),there is no negative and no partial marking provisions. There is no negative marking in Section –C (NAT) as well.

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SECTION-A

[Multiple Choice Questions (MCQ)]

Q. 1 – Q. 10 carry one mark each.

1. Let $A=(a_{ij})_{4\times 3}$ matrix with real entries such that the space of all solutions of the linear system.

$$Ax=\begin{bmatrix}1\\2\\3\\4\end{bmatrix} \text{ is given by } \left\{\begin{bmatrix}1+2t\\2+3t\\3+4t\end{bmatrix}: t\in\mathbb{R}\right\}$$

Then Rank(A) = ?

(a) 4 (b) 3 (c) 2 (d) 1

2. The integrating factor of the differential equation $\frac{dy}{dx}(x\log x)+y=2\log x$ is

(a) e^x (b) $\log x$ (c) $\log(\log x)$ (d) x

3. Let f be a real valued function satisfying $|f(x)-f(a)|\le c\cdot|x-a|^{\gamma}$ for some $\gamma>0$ and $c>0$.

Consider

S_1 : for $\gamma=1$, f is continuous at a.

S_2 : for $\gamma>1$, f is differentiable at a.

Then

(a) Only S_1 is true (b) only S_2 is true (c) Both are true (d) None of them is true

4. Evaluate the tripple integral $\iiint_T y\,dx\,dy\,dz$, where 'T' is the Region bounded by the surfaces $x=y^2$ $x=y+2, 4z=x^2+y^2$ and $z=y+3$.

(a) $\frac{837}{160}$ (b) $\frac{637}{160}$ (c) $\frac{835}{160}$ (d) $\frac{635}{160}$

5. If $f'(x)=g(x)(x-a)^2$ where $g(a)\ne 0$ and g is continuous at $x=a$, then

(a) f is increasing near a if $g(a)<0$

(b) f is decreasing near a if $g(a)>0$

(c) f is increasing near a if $g(a)>0$

(d) None of these

6. Number of elements of order 20 in $U(100)$ are

(a) 8 (b) 12 (c) 16 (d) 20

7. Let $\mathbb{R}[x]$ be the vector space of all real polynomials over field '$\mathbb{R}$'. Which of the following subsets of $\mathbb{R}[x]$ is not a subspace.

(a) $\{p(x): p(1) \geq 0\}$

(b) $\{p(x): p'(0)+p(0)=0\}$; $p'(x)$ is derivative of $p(x)$

(c) $\{p(x): p(x)=p(1-x)\ \forall x\}$

(d) $\{p(x): p(x)$ has a real root at $x = 2\}$

8. Let $y(t)$ be the solution of the initial values problem $y''+y'-2y=0,\ y(0)=b,\ y'(0)=0$, for which value of b is $\lim_{t\to\infty} y(t)=0$?

(a) There is no value for b with the limit 0 (b) 0

(c) –1 (d) 1

9. The angle between the surfaces $x\ln z = y^2-1$ and $x^2 y = 2-z$ at the point (1, 1, 1) is

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) None

10. Let $P(x)$ be a polynomial in the real variable x of degree 7. Then $\lim_{n\to\infty} \frac{P(n)}{2^n}$ is

(a) 5 (b) 7 (c) ∞ (d) 0

Q. 11 – Q. 30 carry two marks each.

11. Let R^+ be multiplications group of all postive reals and R be a addifine group of all reals and $f: R^+ \to R$ be a map given by $f(x)=\log x\ \forall\ x \in R^+$,

(a) f is not a homomorphism (b) f is an isomorphism

(c) f is a homomorphism but not onto (d) f is a homomorphism but not one-one

12. Let 'V' be the vector space of dimension '3' and 'F' be a finite field with '4' elements and let 'V' be the vector space over field 'F' then number of distinct basis of V is

(a) 100 (b) 200 (c) 300 (d) None

13. If $x(t)$ is a solution of $(1-t^2)dx - tx\,dt = dt$ and $x(0)=1$, then $x\left(\frac{1}{2}\right)$ is equal to

(a) $\frac{2}{\sqrt{3}}\left(\frac{\pi}{6}+1\right)$ (b) $\frac{2}{\sqrt{3}}\left(\frac{\pi}{6}-1\right)$ (c) $\frac{\pi}{3\sqrt{3}}$ (d) $\frac{\pi}{\sqrt{3}}$

14. Let $f(x,y)=\begin{cases}\dfrac{xy^2}{x^2+y^2} & ; \ (x,y)\neq(0,0)\\ 0 & ; \ (x,y)=(0,0)\end{cases}$

Consider
[S_1] f has directional derivatives at (0, 0) in every direction.
[S_2] f is differentiable at (0, 0).
Then
(a) Only S_1 is true (b) Only S_2 is true (c) Both S_1 and S_2 is true (d) None of them is true

15. Evaluate the integral $\iint\limits_R (x-y)^2\cos^2(x+y)\,dx\,dy$ where 'R' is rhombus with successive vertices at $(\pi,0),(2\pi,\pi),(\pi,2\pi)$ and $(0,\pi)$.

(a) π^4 (b) $\pi^{4/2}$ (c) $\pi^{4/3}$ (d) $\pi^{4/4}$

16. Let 'A' and 'B' be $n\times n$ real matrices such that AB = BA = 0 and A + B is invertible, then which of the following is not always true?
(a) $rank(A)=rank(B)$ (b) $rank(A)+rank(B)=n$
(c) $nullity(A)+nullity(B)=n$ (d) $A-B$ is invertible

17. Let $f(x)$ be differentiable on the interval $(0,\infty)$ such that $f(1)=1$ and $\lim\limits_{t\to x}\dfrac{t^2 f(x)-x^2 f(t)}{t-x}=1$, for each $x>0$. Then $f(x)$ is

(a) $\dfrac{1}{3x}+\dfrac{2x^2}{3}$ (b) $-\dfrac{1}{3x}+\dfrac{4x^2}{3}$ (c) $-\dfrac{1}{x}+\dfrac{2}{x^2}$ (d) $\dfrac{1}{x}$

18. For a linear transformation $T:\mathbb{R}^{10}\to\mathbb{R}^6$, Kernel of T is $\{(x_1,x_2...x_{10})\in\mathbb{R}^{10};$ $x_4=2x_2=3x_3=4x_4=5x_5; x_6+x_7+x_8+x_9+x_{10}=0\}$, then dimension of Range of 'T' is
(a) 5 (b) 4 (c) 6 (d) 2

19. The line integral of $\vec{v}=x^2\hat{i}-2y\hat{j}+z^2\hat{k}$ over the straight line path from (–1, 2, 3) to (2, 3, 5) is

(a) $\dfrac{62}{3}$ (b) $\dfrac{92}{3}$ (c) 25 (d) None

20. Consider the sequence $\{x_n\}_{n\geq1}$ defined recursivily as $n_1=1$, $x_n=\sup\left\{x\in[0,x_{n-1}):\sin\dfrac{1}{x}=0\right\}$

Then $\lim\sup x_n$ equals
(a) $-\infty$ (b) 0 (c) 1 (d) ∞

21. The solution of the equation $b^{-1}cx^2ac^3=bac$ in K_4 is where K_4 is a abelian group of order 4
(a) a (b) b (c) c (d) None of these

22. $\iint_R [x+y]\, dx\, dy$, over the rectangle $R=[0,1]\times[0,2]$, where [.] is greatest integer function of '.'

(a) 0 (b) 1 (c) 2 (d) 3

23. The directional devivative of $f(x,y,z)=xy^2+4xyz+z^2$ at the point (1, 2, 3) in the direction of $3\hat{i}+4\hat{j}-5\hat{k}$ is

(a) $\frac{2+\sqrt{3}}{2}$ (b) $\frac{5+14\sqrt{3}}{2}$ (c) $\frac{7+15\sqrt{3}}{2}$ (d) None

24. The sum of the series $\sum_{k=1}^{\infty}\frac{k^2}{\lfloor k}$ is

(a) $\log 2$ (b) 1 (c) $2e$ (d) e

25. Number of onto homomorphism from Z_8 to Z_6 is / are

(a) 1 (b) 2 (c) 3 (d) 0

26. Let $A=\{(a_{ij})_{5\times 5}$ s.t. $a_{ij}=i\cdot j^2 \forall i \,\&\, j\}$ then

(a) A is diagonalizable (b) det (A) = 15

(c) A is nilpotant (d) Rank (A) = 5

27. Consider the initial value problem $\frac{dy}{dx}=f(x,y),\ \ y(0)=1$, for which of the following choices of function f should we not expect a unique solution ?

(a) $f(x,y)=x+y$ (b) $f(x,y)=e^{xy+y}$ (c) $f(x,y)=y^{1/3}$ (d) $f(x,y)=(1-y)^{4/5}$

28. Let $f(x,y)=\begin{cases} xy\dfrac{x^4-y^4}{x^4+y^4} & \text{if} \quad (x,y)\neq(0,0) \\ 0 & \text{if} \quad (x,y)=(0,0)\end{cases}$. Which of the following is true ?

(a) $f_{xy}(0,0)\neq f_{yx}(0,0)$ (b) $f_{xy}(x,y)=f_{yx}(x,y)\ \forall\,(x,y)\in\mathbb{R}^2$

(c) $f_x(h,0)$ does not exist for any real h (d) $\lim_{(x,y)\to(0,0)} f(x,y)=1$

29. For the set $\left[\frac{1}{n}+\frac{1}{m};\, m,n\in\mathbb{N}\right]$, the element $\frac{1}{u}$ where $u\in\mathbb{N}$ is

(a) Both an element in the set and a limit point of the set.
(b) Neither an element in the set nor a limit point.
(c) An element in the set but not a limit point.
(d) A limit point of the set, but not an element in the set.

30. The arc of the parabola $y=x^2$ from (1, 1) to (2, 4) is rotated above the y-axis. The area of resulting surface is

(a) $\frac{\pi}{3}\left(17\sqrt{17}-5\sqrt{5}\right)$ (b) $\frac{\pi}{6}\left(17\sqrt{17}-5\sqrt{5}\right)$ (c) $\frac{\pi}{2}\left(17\sqrt{17}-5\sqrt{5}\right)$ (d) $\frac{\pi}{4}\left(17\sqrt{17}-5\sqrt{5}\right)$

SECTION-B

[Multiple Select Questions (MSQ)]

Q. 31 – Q. 40 carry two marks each.

31. If $f(x)=[x]+\left[x+\frac{1}{3}\right]+\left[x+\frac{2}{3}\right]$, then ([.] denotes the greatest integer function)

(a) $f(x)$ is discontinuous at $x=1,\ 10,\ 15$

(b) $f(x)$ is continuous at $x=\frac{n}{3}$, where n is any integer

(c) $\int_0^{2/3} f(x)dx=\frac{1}{3}$

(d) $\lim_{x\to\frac{2}{3}} f(x)=2$

32. Let f and g be two functions defined on an Interval I. Then the maximum and minimum function of f and g defined for all $x\in I$ by $\max(f,g)(x)=\max\{f(x),g(x)\}$

and $\min(f,g)(x)=\min\{f(x),g(x)\}$ respectively. Then which of the following statement is/are True?

(a) $\max(f,g)+\min(f,g)=f+g$ for all $x\in I$.

(b) $\max(f+h,g+h)=\max(f,g)+h$ for any function h defined on I.

(c) $\min(f+h,g+h)=\min(f,g)+h$ for any function h defined on I.

(d) $\min(f,g)\le\max(f,g)$ for all $x\in I$.

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33. $$f(x)=\begin{cases}0 & ; \quad if\ x\in Q^c \text{ or } x=0\\ \frac{1}{q} & ; \quad if\ x=\frac{p}{q},\ p\in\mathbb{Z},\ q\in\mathbb{N} \text{ and } p,\ q \text{ are co-prime}\end{cases}$$

Then,

(a) $f(x)$ is continuous on [0, 1].

(b) $f(x)$ is continuous only at irrationals.

(c) $f(x)$ is discontinuous only at rationals.

(d) $f(x)$ is integrable on [0, 1] and $\int_0^1 f(x)\,dx=0$.

34. Let $T:\mathbb{R}^4 \to \mathbb{R}^3$ be the linear tranformation given by

$T(x,y,z,t) = (x-y+z+t, x+2z-t, x+y+3z-3t)$

then

(a) $\{(1,1,1),(0,1,2)\}$ form a basis for Range space of 'T'

(b) $\{(2,1,-1,0),(1,2,0,1)\}$ form a basis of Ker 'T'

(c) Rank T = 2

(d) Nullity T = 2

35. $f(x) = [x] + (x-[x])^{[x]}$ for $x \geq \frac{1}{2}$, then

(a) $f(x)$ is continuous.

(b) $f(x)$ is strictly increasing on $[1,\infty)$.

(c) $f(x)$ is integrable on every closed subset of $\left[\frac{1}{2},\infty\right)$.

(d) None of the above.

36. Let $a_n = 1+\frac{1}{2}+\frac{1}{4}+\ldots+\frac{1}{n-1}-\log n$ and

$b_n = 1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}-\log n$ for all $n \geq 2$

then,

(a) Both $\langle a_n \rangle$ and $\langle b_n \rangle$ are convergent in $\mathbb{R}$.

(b) Both $\langle a_n \rangle$ and $\langle b_n \rangle$ are divergent in $\mathbb{R}$.

(c) $\langle a_n \rangle$ is increasing and $\langle b_n \rangle$ is a decreasing sequence.

(d) Both $\langle a_n \rangle$ and $\langle b_n \rangle$ are increasing.

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37. Choose incorrect statements:

(a) There is an isomorphism from $(Q,+)$ to $(Q^*,\cdot)$ where $Q^* = Q-\{0\}$

(b) $Aut(G_1) \cong Aut(G_2)$ and G_1 is infinite group then G_2 is also infinite.

(c) If $Aut(G_1) \cong Aut(G_2)$ and G_1 is finite then G_2 is finite

(d) $G_1 \not\cong G_2$ then $Aut(G_1) \not\cong Aut(G_2)$

(where symbol $\cong$ denote isomorphism)

38. Let $f:\mathbb{R}\to\mathbb{R}$ be a continuous function with period $p>0$. Then $g(x) = \int_x^{x+p} f(t)\,dt$ is a

(a) constant function
(b) continuous function
(c) continuous function but not differentiable
(d) neither continuous nor differentiable

39. Let the vector field $\vec{F} = (ay^2 + 2czx)\hat{i} + y(bx + cz)\hat{j} + (ay^2 + cx^2)\hat{k}$. The integral $\oint_C \vec{F} \cdot d\vec{r} = 0$ for any piecewise smooth closed curve C if

(a) $b = c = 2a$ (b) $a = c = 2b$ (c) $y = 0$ (d) $z = 0$

40. Let us consider a vector field $\vec{F} = 2x\left(y^2 + z^2\right)\hat{i} + 2x^2y\,\hat{j} + 3x^2z^2\,\hat{k}$

Then,

(a) $\vec{F}$ is conservative
(b) The total work done in moving a particle along a closed path is '0'.
(c) The work done in moving a particle from (–1, 2, 1) to (2, 3, 4) is 287.
(d) None of the above.

SECTION-C

[Numerical Answer Type (NAT)]

Q. 41 – Q. 50 carry one mark each.

41. The work done by the force $\vec{F} = \left(x^2 - y^3\right)\hat{i} + \left(x + y\right)\hat{j}$ in moving a particle along a closed path 'c' containing the curves $x + y = 0, x^2 + y^2 = 16$ and $y = x$ in the first and fourth quadrants is ________________.

42. Let $\omega = \log\left(u^2 + v^2\right)$ where $u = e^{\left(x^2 + y\right)}$ and $v = e^{\left(x + y^2\right)}$. Then $\left.\frac{\partial\theta}{\partial x}\right|_{x=0, y=0}$ is ________________.

43. The number of non-isomorphic groups of order 6 is ________________.

44. Evaluate $\iint_D \left(x + 2y\right) dA$, where '$D$' is the region bounded by $y = 2x^2$ and $y = 1 + x^2$ is ________________.

45. $\lim\limits_{(x,y)\to(0,0)} \frac{\cos xy - 1}{x} =$ ________________

46. Let $\vec{F} = x^2z\,\hat{i} + y\,\hat{j} - xz^2\,\hat{k}$ and 'S' is the boundry of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4y$ then $\iint_S \vec{F} \cdot \hat{n}\, ds =$ ________________ $\times \pi$

47. If $x\,dy = y(dx + y\,dy)$, $y(1) = 1$ and $y(x) > 0$. Then, $y(-3)$ is equal to ________________

48. Let $A=\begin{bmatrix}0&0&0&-4\\1&0&0&0\\0&1&0&5\\0&0&1&0\end{bmatrix}$, then trace A^4 is ________________________

49. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{(\sec^2 x-2\tan x)}{1+\cos 4x}$ is ________________.

50. Let $P(x)$ be a non-zero polynomial of degree n. The radius of convergence of the power series $\sum\limits_{n=0}^{\infty}P(n)x^n$ is __________________.

Q. 51 – Q. 60 carry two marks each.

51. The volume of the solid that lies above the cone $z=\sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=z$ is ________________ $\times\ \pi$.

52. Let H_{10} be the space of 10×10 matrices with entries in $\mathbb{R}$ satisfying $a_{ij}=a_{rs}$ whenever $i+j=r+s$ then $\dim H_{10}$ is _________________

53. Let $y=\phi(x)$ and $y=\psi(x)$ be solutions of $y''-2xy'+(\sin x^2)y=0$ such that $\phi(0)=1,\ \phi'(0)=1$ and $\psi(0)=1,\ \psi'(0)=2$. Then the value of Wronskian $w(\phi,\psi)$ at $x=1$ is _______________

54. In a non-abelian group the elements a has order 108 then the order of a^{42} is ________________

55. Let $f:\mathbb{R}\to\mathbb{R}$ be a continuous function s.t. $f(\mathbb{R})\subseteq\mathbb{N}$, if $f(2)=3$ then $f(4)=$ ________________ .

56. Given that $y(x)$ is a solution of the differential equation $\dfrac{dy}{dx}=5y(4-y)$ with $y(0)=2$. When $x\to\infty$ then the solution $y(x)$ increases to α, where α is __________________.

57. Find $\lim(n!)^{1/n^2}=$ ________________ .

58. Let 'S' be the set of non-null diagonalizable matrices such that $A^k=0$ for positive integer 'k' $(2\le k\le n)$, then number of elements is set 'S' is ______________

59. Let 'c' be the trace of the cone $z=\sqrt{x^2+y^2}$ intersected by the plane z = 4 and 'S' in the surface of the cone below z = 4, then $\oint_c 2y^3\,dx+x^3\,dy+z\,dz=$ ________________ $\times\ \pi$.

60. The smallest number n for which $\exists$ a group of order n for which Lagrange's theorem is not true is ________________ .

END OF THE QUESTION PAPER

SPACE FOR ROUGH WORK

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IIT-JAM MATHEMATICS

TEST SERIES - 6
(Full Length Test Series - 3)

Time : 3 Hours **Date : 28-01-2018**
M.M. : 100

ANSWER KEY

SECTION-A

[Multiple Choice Questions (MCQ)]

1. (c)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (c)
6. (c)	**7.** (a)	**8.** (c)	**9.** (d)	**10.** (d)
11. (b)	**12.** (d)	**13.** (a)	**14.** (a)	**15.** (c)
16. (a)	**17.** (a)	**18.** (a)	**19.** (b)	**20.** (b)
21. (d)	**22.** (c)	**23.** (d)	**24.** (c)	**25.** (d)
26. (a)	**27.** (d)	**28.** (a)	**29.** (a)	**30.** (b)

SECTION-B

[Multiple Select Questions (MSQ)]

31. (a, c)	**32.** (a,b,c,d)	**33.** (b,c,d)	**34.** (a,b,c,d)
35. (a,b,c)	**36.** (a,c)	**37.** (a,b)	**38.** (a,b)
39. (a,c)	**40.** (a,b,c)		

SECTION-C

[Numerical Answer Type (NAT)]

41. (67.28)	**42.** (1)	**43.** (2)	**44.** (2.13)	**45.** (0)
46. (8)	**47.** (3)	**48.** (34)	**49.** (0.5)	**50.** (1)
51. (0.125)	**52.** (19)	**53.** (2.7182)	**54.** (18)	**55.** (3)
56. (4)	**57.** (1)	**58.** (0)	**59.** (192)	**60.** (12)