D.U. M.Sc. MATHEMATICS ENTRANCE - 2017

D.U. Entrance Test - 2017

1. The sequence ($n^{1/n}$) is

(a) monotonically decreasing
(b) monotonically increasing
(c) convergent and converges to zero
(d) neigher monotonically increasesing or monotonically decreasing

2. Let $S=\prod_{n=1}^{\infty}\left[-\frac{1}{n},1+\frac{1}{hn}\right]$ the S equals

(a) [0, 1] (b) (0, 1] (c) (0, 1) (d) [0. 1)

3. Consider the series $\sum_{n=1}^{\infty}\frac{(-1)^{n/1}}{n}\left(\sqrt{n+1}-\sqrt{n-1}\right)$ then

(a) the series is convergent but not absolutely convergent
(b) the series is divergent
(c) The nth term of series does not converge to zero
(d) The series is aboslutely convergent

4. Consider the sets $S=\left\{\frac{1}{n};n\in\mathbb{N}\text{ and }n\text{ is prime}\right\}$ $T=\left\{x^2:x\in\mathbb{R}\right\}$ Then

(a) sup (S∩T) = 1
(b) sup S = 1 and inf T = 0
(c) Sup $S=\frac{1}{2}$ and inf T = 0
(d) inf (S ∪ T) $\frac{1}{2}$

5. Consider the following functions from $\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ defined by

$$d_1(x,y)=|x|+|y|,$$

$$d_2(x,y)=\begin{cases}2, & x\neq y\\ 0, & x=0\end{cases}$$

$d_3(x,y)=\sqrt{|x-y|}$. Which of the following statements is true ?

(a) Only d_2 and d_3 are metrics on $\mathbb{R}$
(b) Only d_3 is metric on $\mathbb{R}$
(c) Only d_1 and d_2 are metrics on $\mathbb{R}$
(d) All are metric on $\mathbb{R}$

6. $S = \{(x, y) \in \mathbb{R}^2 : xy < 0\}$

(a) neither connected nor compact subset of $\mathbb{R}^2$
(b) not connected nor compact subset of $\mathbb{R}^2$
(c) is both connected and compact subset of $\mathbb{R}^2$
(d) is not compact subset of $\mathbb{R}^2$ but connected

7. Let (x_n) be a sequence defined by :

$x_1 = 3$ and $x_{n+1} = \dfrac{1}{4 - x_n}$ Then

(a) (x_n) is a monotonically decreasing sequence that is not boundedc below
(b) (x_n) converges to $2+\sqrt{3}$
(c) (x_n) converges to $2-\sqrt{3}$
(d) (x_n) diverges

8. The value of the series $\sum_{n=1}^{\infty} \dfrac{n}{2^n}$ is given by

(a) 2
(b) 4
(c) 6
(d) 8

9. Let f be a continous funtion on $\mathbb{R}$. Define $G(x) = \int_0^{\sin x} f(t)\,dt \quad \forall x \in \mathbb{R}$. Then

(a) $G'(x) = f(\cos x) \sin x$
(b) $G'(x) = -f(\sin x) \cos x$
(c) $G'(x) = f(\sin x) \cos x$
(d) $G'(x) = f(\sin x) \sin x$

10. Let (X,d) be a metric space wher X is an infinite set and d is the discrete metric. Then

(a) Heine- Borel theorem holds for (X,d)
(b) Heine- Borel theorem does not holds for (X,d)
(c) X is not bounded
(d) X is compact

11. Let $f_n(x) = \dfrac{1}{1+(nx-1)^2}, x \in [0,1]$ Then the sequence (f_n) is

(a) pointwise convergent but not uniformaly convergent on [0, 1]
(b) uniformaly convergent but not Pointwise convergent on [0, 1]
(c) both pointwise and uniformaly convergent on [0, 1]
(d) neither pointwise and uniformaly convergent on [0, 1]

12. The limit inferior of the sequence (x_n) where $x_n = 1 + (-1)^n + \dfrac{1}{3^n}$ is

(a) 1
(b) 3
(c) 2
(d) 0

13. Which of the following sets in in one - to - one correspondence with $\mathbb{N}$

(I) $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\ldots\ldots\right\}$

(II) $\{\ldots\ldots\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\ldots\}$

(III) $\left\{\frac{p}{q}; p, q \in \mathbb{Z}, q \neq 0\right\}$

(IV $\left\{\frac{p}{q}; p, q \in \mathbb{N}\right\}$

(a) (I) and (II)
(b) (I), (II) and (III)
(c) (I) and (IV)
(d) All of the above

14. Suppose f and g are differentiable on the interval [a, ∞) such that $f(a) \le g(a)$ and $f'(x) < g'(x) \forall x > a$. Then which of the following statements is true ?

(a) $f(x) = g(x) \alpha \forall x \in [a, \infty)$ (b) $f(x) > g(x)$

(c) $f(x) < g(x)$ (d) None of the above

15. Which of the following statements are true ?

(I) There exists as continuous function from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ onto (0, 1)

(II) There exists continuous function from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ onto $\mathbb{R}$

(III) There exists a continuous function from $[0, \pi] \cup [2\pi, 3\pi]$ onto [0, 1]

(IV) There exists a continuous function from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ onto $\left[0, \frac{1}{3}\right] \cup \left[\frac{2}{3}, 1\right]$

(a) (I) and (II) (b) (II) and (III) (c) (III) and (IV) (d) (I) and (IV)

16. For $x = (x_1, x_2 x_3), = (y_1, y_2, y_3) \in \mathbb{R}^3$

$$d_1(x, y) = \max_{1 \le j \le 3} | x_i - y_j |$$

$$d_2(x, y) = \left[\sum_{j=1}^{3} (x_j - y_j)^2 \right]^{1/2}$$

Consider the metric spaces $(\mathbb{R}^3, d_1)$ and $(\mathbb{R}^3, d_2)$ then

(a) $(\mathbb{R}^3, d_1)$ is complete, but $(\mathbb{R}^3, d_2)$ is not complete

(b) $(\mathbb{R}^3, d_2)$ is complete, but $(\mathbb{R}^3, d_1)$ is not complete

(c) Both $(\mathbb{R}^3, d_1)$ and $(\mathbb{R}^3, d_2)$ are complete

(d) Neither $(\mathbb{R}^3, d_1)$ nor $(\mathbb{R}^3, d_2)$ is complete

17. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = \begin{cases} \frac{x^2 y}{x^4 + y^2} & , (x, y) \ne (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$ Then

(a) f is not countinuous at (0, 0) but all directional derivatives of f at (0, 0) exist

(b) f is countinuous in $\mathbb{R}^2$ and all directional derivatives at (0, 0) exist

(c) f is countinuous in $\mathbb{R}^2$ but not all directional derivatives at (0, 0) exist

(d) f is not countinuous at (0, 0) and not all directional derivatives at (0, 0) exist.

18. Let $X = \{(x, y) \in \mathbb{R}^2 : x \in \mathbb{Q}, y \in \mathbb{R} \setminus \mathbb{Q}\}$ where $\mathbb{Q}$ is the set of rationals. Then

(a) X is an opne and dense subset of $\mathbb{R}^2$
(b) X is an opne and dense subset of $\mathbb{R}^2$
(c) X is not an open but a dense subset of $\mathbb{R}^2$
(d) X is neither an open nor a dense subset of $\mathbb{R}^2$

19. Let $n \in \mathbb{N}, n \geq 3$ be fixed and let $f : [0, 1] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x & 0 \leq x \leq 1/n \\ x - \dfrac{(2k-1)}{2n} & \dfrac{k-1}{n} < x \leq \dfrac{k}{n} \\ & k = 2, 3, \ldots\ldots\ldots, n \end{cases}$$ then

(a) f is continuous and Riemann intergrable on [0, 1]
(b) f is not continuous but is Riemann intergrable on [0, 1]
(c) f is continuous but not Riemann intergrable on [0, 1]
(d) f is neither continuous nor Riemann intergrable on [0, 1]

20. Let $S = \{x \in \mathbb{R} : 3 - x^2 > 0\}$ Then

(a) S is bounded above and 3 is the least upper bound of S.
(b) S is bounded above and does not have a least upper bound of $\mathbb{R}$.
(c) S is bounded above and does not have a least upper bound in $\mathbb{Q}$ the set of rational numbers
(d) S is not bounded above

21. Let p and q be distinct primes and let G and H be two graphs such that $0(G) = p$ and $0(H) = p$. The number of distinct homomorphims from G to H is/ are

(a) 1 (b) p –1 (c) q –1 (d) pq

22. Let G be a cyclic group such that G has an element of infinite order. Then the number of elements of finite order in G is/are

(a) 0 (b) 1 (c) infinity (d) none of these

23. Let G be non-abelien group of order p3 where p is a prime. Let $\mathbb{Z}(G) \neq \{e\}$. Then

(a) $0(\mathbb{Z}(G)) = p$ (b) $0(\mathbb{Z}(G)) = p^2$ (c) $\dfrac{G}{\mathbb{Z}(G)}$ is cyclic (d) none of the above

24. Let G be a group of order pqr, where p,q,r are primes and p <q < r. Which of the following statements are true ?

(i) G has a normal subgroup of order qr
(ii) Sylow r-subgroup of G is normal
(iii) G is abelian

(a) Only (i) and (ii) (b) Only (ii) and (iii) (c) Only (i) and (iii) (d) (i), (ii) and (iii)

25. Let R be a ring with unity such that each element of R is an idempotent. Then the characteristic of R is

(a) 0 (b) 2 (c) an odd prime (d) none of the above

26. Let $F = \mathbb{Q}(\sqrt{2i})$ Which one of the following is not ture ?

(a) $\sqrt{2} \in F$
(b) $i \in \mathbf{F}$
(c) $x^8 - 16 = 0$ has a solution in F
(d) $\dim_{\mathbb{Q}}(F) = 2$

27. The ideal $\langle x\rangle$ of the ring $\mathbb{Z}[x]$ is

(a) maximal but not prime (b) prime but not maximal

(c) both prime and maximal (d) neither prime nor maximal

28. The smallest subring of $\mathbb{Q}$ containing $\frac{2}{3}$ is

(a) $S=\left\{a+b\frac{2}{3}\mid a,b\in\mathbb{Z}\right\}$ (b) $S=\mathbb{Q}$

(c) $S=\left\{a\left(\frac{2}{3}\right)^k\mid k\in\mathbb{N}, a\in\mathbb{Z}\right\}$

(d) $S=\left\{a_0+a_1\frac{2}{3}+a_2\left(\frac{2}{3}\right)^2+.........+a_n\left(\frac{2}{3}\right)^n\mid n\in\mathbb{N}, a_0, a_1.......a_n\in\mathbb{Z}\right\}$

29. If p is an odd prime, then

$\phi(p)+\phi(2p)+\phi(2^2 p)+.........+\phi(2^m p)$ is equal to

(a) $(2^m-1)(p-1)$ (b) $2^m(p-1)$

(c) $(2^m+1)(p-1)$ (d) $2^{m+1}(p-1)$

30. Let $A(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}, \theta\in(0,2\pi)$

Which of the following statements is ture ?

(a) A (θ) has eigenvectors in $\mathbb{R}^2$ for every $\theta\in(0,2\pi)$

(b) A (θ) does not have eigenvectors in $\mathbb{R}^2$ for any $\theta\in(0,2\pi)$

(c) A (θ) has eigenvectors in $\mathbb{R}^2$ for exactly one value of $\theta\in(0,2\pi)$

(d) A (θ) has eigenvectors in $\mathbb{R}^2$ for exactly two value of $\theta\in(0,2\pi)$

31. Let **M** (n, $\mathbb{R}$) be the vector space of n×n matrices with real entries and U be the subset of M (n, $\mathbb{R}$) given by $\{(a_{ij})\mid a_{11}+a_{22}+.......+a_{nn}=0\}$ Which one of the following statements is true ?

(a) U is a subspace of dimention n^2-1 (b) U is a subspace of dimension n^2-n

(c) U is not a subspace (d) None of the above

32. Let $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ Then det $(A^3-6A^2+5A+3I)$ is

(a) 24 (b) 15 (c) 3 (d) 0

33. Let $V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} a, b, c, d \in \mathbb{R} \right\}$ and $W = \left\{ a + bx + cx^2 \mid a, b, c \in \mathbb{R} \right\}$ define T : V→W by

$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+b) + (b-c)x + (x+d)x^2$ The null space of T is

(a) $\left\{ a \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} a \in \mathbb{R} \right\}$ (b) $\left\{ a \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} a \in \mathbb{R} \right\}$

(c) $\left\{ a \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} a \in \mathbb{R} \right\}$ (d) $\left\{ a \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} a \in \mathbb{R} \right\}$

34. Let $W_1 = \left\{ (a, 2a, 0) \mid a \in \mathbb{R} \right\}$

$W_2 = \left\{ (a, 0, -a) \mid a \in \mathbb{R} \right\}$ Then

(a) $W_1 + W_2$ is a subspace of $\mathbb{R}^3$ but $W_1 \cup W_2$ is not

(b) $W_1 + W_2$, $W_1 \cup W_2$ are both subspace of $\mathbb{R}^3$

(c) neither $W_1 + W_2$ nor $W_1 \cup W_2$ is a subspace of $\mathbb{R}^3$

(d) $W_1 \cup W_2$ is a subspace of $\mathbb{R}^3$ but $W_1 + W_2$ is not

35. Let V = C [0, π] be an inner product space with inner product

$\langle f, g \rangle = \int_0^{\pi} f(x)\, dx$

Let $f(x)$ = cosx, g(x) = sinx. Then

(a) f, g are orthogonal but linearly independent (b) f, g are orthogonal but linearly independent

(c) f, g are linearly independent but not orthogonal (d) neither f, g are linearly indepedent nor orthogonal

36. If the partial differential equation

$$(x-2)^2 \frac{\partial^2 u}{dx^2} - (y-3)^2 \frac{\partial^2 u}{\partial y^2} + 2x\frac{\partial u}{\partial x} + y + \frac{\partial u}{\partial y} = u$$

is parabolic in the region $S \subseteq \mathbb{R}^2$ but not in $\mathbb{R}^2 \setminus S$. then S is

(a) $\left\{ (x, y) \in \mathbb{R}^2; x = 2 \text{ or } y = 3 \right\}$ (b) $\left\{ (x, y) \in \mathbb{R}^2; x = 2 \text{ or } y = 3 \right\}$

(c) $\left\{ (x, y) \in \mathbb{R}^2; x = 2 \right\}$ (d) $\left\{ (x, y) \in \mathbb{R}^2; x = 3 \right\}$

37. Let $u(x, y)$ be the solution of the Cauchy problem

$$x^2 \frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} - 0$$

$u \to e^x$ as $y \to \infty$ Then $u(1, 1)$

(a) -1 (b) 0 (c) 1 (d) e^{-2}

38. The initial value problem $x\frac{dy}{dx} = 2y$, $y(a) = b$ has

(a) infinitely many solutions through $(0, b)$ if $b \neq 0$

(b) unique solution for all a and b

(c) no solution if $a = b = 0$

(d) infinitely many solutions if $a = b = 0$

39. The solution of the differential equation

$\frac{d^2y}{dx^2} + 4y = \cos 2x$ is given by

(a) $c_1 \cos 2x + c_2 \sin 2x + \frac{x}{4}\sin 2x$

(b) $c_1 \cos 2x + c_2 \sin 2x + \frac{x}{2}\sin 2x$

(c) $c_1 \cos 2x + c_2 \sin 2x + \frac{x}{4}\cos 2x$

(d) $c_1 \cos 2x + c_2 \sin 2x + \cos 2x$

40 The following initial value problem of a first ordwer linear system

$x' = 3x - 2y,\ x(0) = 1$

$y' = -3x + 4y,\ y(0) = -2$ can be converted into an initial vale problem of a 2nd order differential equaation for $x(t)$. It is

(a) $x'' = -7x' + 6x = 0\,;\ x(0) = 1,\ x'(0) = -2$

(b) $x'' = -7x' + 6x = 0;\ x(0) = 1,\ x'(0) = 0$

(c) $x'' = -7x' + 6x = 0;\ x(0) = 1,\ x'(0) = 7$

(d) $x'' - x' + 6x = 0;\ x(0) = 1,\ x'(0) = -2$

41. The characteristic values of the sturm-Lioville problem

$\frac{d^2y}{dx^2} + \lambda t = 0\,;\ y(0) = 0;\ y(\pi) - y'(\pi) = 0,$

are

(a) $\lambda = \alpha_n^2$ where $\alpha_n\,(n = 1, 2, 3..........)$ are the positive roots of equation $\alpha = \cot \pi \alpha$

(b) $\lambda = \alpha_n^2$ where $\alpha_n\,(n = 1, 2, 3..........)$ are roots of the equation $\alpha = \tan \pi \alpha$

(c) 0, 1]

(d) negative real numbers

42. Determine an interval in which the solution of the following initial value problem is certain to exist

$y' + (\tan t)\,y = \sin t,\ y(\pi) = 0$

(a) $\frac{\pi}{2} < t < \frac{3\pi}{2}$

(b) $0 < t < \frac{3\pi}{2}$

(c) $\frac{\pi}{2} < t < 6$

(d) $0 < t < 3\pi$

43. The derivative $\frac{du}{dx}$ can be approximated most accurately by which finite difference

(a) $\frac{v_{k+1}^n - v_k^n}{\Delta x}$ (b) $\frac{v_k^n - v_{k-1}^n}{\Delta x}$

(c) $\frac{v_{k+1}^n - v_{k-1}^n}{2\Delta x}$ (d) All are equally accurate

44. What are the solution α if any, of the equation $x = \sqrt{1+x}$? Does the iteration $x_{n+1} = \sqrt{1+x_n}$ converge to any of these solutions ?

(a) Root $= \frac{1+\sqrt{5}}{2}$, iterations converge with $x_0 = 1$ (b) Root $= \frac{1-\sqrt{5}}{2}$, iterations converge with $x_0 = -1$

(c) Both (A) and (B)

(d) Roots $= \frac{1 \pm \sqrt{5}}{2}$ but the iterations do not converge to any root

45. Is the following function a cubic spline on the interval $0 \le x \le 2$

$$s(x) = \begin{cases} (x-1)^3 & , 0 \le x \le 1 \\ 2(x-1)^3 & , 1 \le x \le 2 \end{cases}$$

(a) Yes, it is a cubic spline on [0, 2] (b) It is a cubic spline only on [0, 1]

(c) It is a cubic spline only on [0, 1] (d) It is not a cubic spline

46. Consider the second order differential equation $x^2 y''(x) + xy'(x) - 9y(x) = 0$ for $x > 0$ If the solution satisfies the initial conditions $y(1) = 0,\ y'(1) = 2$, then y (2) is

(a) $\frac{21}{8}$ (b) $\frac{63}{8}$ (c) $\frac{7}{16}$ (d) $\frac{63}{4}$

47. The eigenvalues associated with the BCP $y''(x) - 2y'(x) + (1-\lambda)y(x) = 0$ y (0) = 0, y (1) = 0 is /are

(a) $\lambda = 0$ (b) $\lambda = \pi^2 n^2,\ n = 1, 2, 3, \ldots\ldots.$

(c) $\lambda = -\pi^2 n^2,\ n = 1, 2, 3, \ldots\ldots.$ (d) $\lambda = -\pi n,\ n = 1, 2, 3, \ldots\ldots.$

48. The value of $I = \int_0^{\sqrt{n}} \sin x^2 dx$ using the trapezium rule with two subintervals is

(a) $\frac{\pi}{4}$ (b) $\frac{\sqrt{\pi}}{4}$ (c) $\frac{\sqrt{\pi}}{2}$ (d) $\frac{\sqrt{2\pi}}{4}$

49. Consider the system of equations $\begin{bmatrix} 1 & -a \\ -a & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ where 'a' is a constant Gauss- Seidel method for the solution of the above system converges for

(a) All values of a (b) $|a| < 1$ (c) $|a| > 1$ (d) $a > 2$

50. The error in the value of y at 0.2 when modified Euler's method is used to solve the problem

$\frac{dy}{dx} = x - y(0) = 1,\ h = 0.2$ is of the order

(a) 10^{-1} (b) 10^{-2} (c) 10^{-3} (d) 0

D.U. M.Sc. MATHEMATICS ENTRANCE - 2017

ANSWER KEY

1. (d)	2. (a)	3. (d)	4. (c)	5. (a)
6. (a)	7. (c)	8. (a)	9. (c)	10. (b)
11. (a)	12. (c)	13. (d)	14. (c)	15. (b)
16. (c)	17. (a)	18. (c)	19. (b)	20. (c)
21. (a)	22. (b)	23. (a)	24. (d)	25. (b)
26. (a)	27. (b)	28. (d)	29. (b)	30. (c)
31. (a)	32. (c)	33. (a)	34. (a)	35. (b)
36. (a)	37. (d)	38. (d)	39. (a)	40. (c)
41. (b)	42. (a)	43. (c)	44. (a)	45. (a)
46. (a)	47. (c)	48. (d)	49. (b)	50. (c)