D.U. M.Sc. MATHEMATICS ENTRANCE - 2018

1. The complete integral of the partial differential equation $xpq + yq^2 - 1 = 0$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$ is

   (a) $(z+b)^2 = 4(ax+y)$ (b) $z+b = 2(ax+y)$

   (c) $z+b = 4(ax+y)^2$ (d) $z+b = 2(ax+y)^2$

2. Let P be the set of all the polynomials with rational coefficients and S be the set of all sequences of natural numbers. Then which one of the following statements is true ?

   (a) S is countable but P is not (b) Both the sets P and S are uncountable.

   (c) Both the sets P and S are countable (d) P is countable but S is not

3. For the differential equation $x\frac{dy}{dx} + 6y = 3xy^{4/3}$, consider the following statements :

   (i) The given differential equation is a linear equation.

   (ii) The differential equation can be reduced to linear equation by transformation $V = y^{-1/3}$.

   (iii) The differential equation can be reduced to linear equation by transformation $V = x^{-1/3}$

   Which of the above statements are true ?

   (a) Only (i) (b) Only (iii)

   (c) Only (ii) (d) Both (i) and (ii)

4. Which one of the following statements is not true for Simpson's 1/3 rule to find approximate value of the difinite itnegral $I = \int_0^1 f(x)\,dx$?

   (a) If $y_0 = f(0), y_1 = f(0.5), y_2 = f(1)$, the approximate value of I is $\frac{1}{6}[y_0 + 3y_1 + y_2]$.

   (b) The approximating function has odd number of points common with the function $f(x)$.

   (c) Simpson's $\frac{1}{3}$ rule improves trapezoidal rule.

   (d) The function $f(x)$ is approximated by a parabola.

5. The equation of the tangent plane to the surface $z = 2x^2 - y^2$ at the point (1, 1, 1) is

   (a) $x - y - 2z = 2$ (b) $4x - y - 3z = 1$

   (c) $2x - y - 2z = 1$ (d) $4x - 2y - z = 1$

6. If $\{x, y\}$ is an orthonormal set in an inner product space then the value of $\| x - y \| + \| x + y \|$ is

(a) $2\sqrt{2}$ (b) $2 + \sqrt{2}$

(c) $\sqrt{2}$ (d) 2

7. Which one of the followig spaces, with the usual metric, is not separable?

(a) The space $C[a, b]$ of the set of all real valued continuous functions defined on $[a, b]$.

(b) The space l^∞ of all bounded real sequences with supremum metric.

(c) The Euclidean space $\mathbb{R}^n$.

(d) The space l^1 of all absolutely convergent real sequences.

8. Let G be an abselian group of roder 2018 and $f : G \to G$ be defined as $f(x) = x^5$. Then

(a) f is not injective (b) f is not surjective

(c) there exsits $e \neq x \in G$ such that $f(x) = x^{-1}$ (d) f is an automorphism of G

9. If $f : \mathbb{R} \to \mathbb{R}$ is continuous function such that $f(x + y) = f(x) + f(y)$, for all $x, y \in \mathbb{R}$, then

(a) f is increasing if $f(1) \geq 0$ and decreasing if $f(1) \leq 0$.

(b) f is increasing if $f(1) \leq 0$ and decreasing if $f(1) \geq 0$.

(c) f is a not an increasing function.

(d) f is neither an increasing nor a decreasing function.

10. The central difference operator δ and backward difference operator ∇ are related as

(a) $\delta = \nabla(1 - \nabla)^{\frac{1}{2}}$ (b) $\delta = \nabla(1 + \nabla)^{-\frac{1}{2}}$

(c) $\delta = \nabla(1 - \nabla)^{-\frac{1}{2}}$ (d) $\delta = \nabla(1 + \nabla)^{\frac{1}{2}}$

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11. How many continuous real functions f can be defined on $\mathbb{R}$ such that $(f(x))^2 = x^2$ for every $x \in \mathbb{R}$

(a) Infinitely many (b) None

(c) 4 (d) 2

12. The greatest common divisor of $11 + 7i$ and $18 - i$ in the ring of Gaussian integers $\mathbb{Z}[i]$ is

(a) $3i$ (b) 1

(c) $1 + i$ (d) $2 + i$

13. The complete integral of the partial differential equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{x+2y}$ is

(a) $\phi_1(y - x) + x\phi_2(y + x) + e^{x+2y}$ (b) $\phi_1(y + x) + x\phi_2(y + x) + xe^{x+2y}$

(c) $\phi_1(y - x) + \phi_2(y + x) + e^{x+2y}$ (d) $\phi_1(y + x) + x\phi_2(y + x) + e^{x+2y}$

14. If $S = \{(1,0,i),(1,2,1)\} \subseteq \mathbb{C}^3$, then $S^{\perp}$ is

(a) span $\left\{\left(i, -\frac{1}{2}(i+1), -1\right)\right\}$ (b) span $\left\{\left(-i, \frac{1}{2}(i+1), 1\right)\right\}$

(c) span $\left\{\left(i, -\frac{1}{2}(i+1), 1\right)\right\}$ (d) span $\left\{\left(i, \frac{1}{2}(i+1), -1\right)\right\}$

15. The improper integral $\int_{-\infty}^{0} 2^x \, dx$

(a) convergent and converges to 2 (b) divergent

(c) convergent and converges to $\frac{1}{\ln 2}$ (d) convergent and converges to -ln2

16. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function which takes irrational values at rational points and rational values at irrational points. Then which one of the following statements is true ?

(a) f is uniformly continuous on $\mathbb{Q}$ (b) f is uniformly continuous on $\mathbb{R}$

(c) f is uniformly continuous on $\mathbb{Q}^c$ (d) No such function exists

17. If $f : [0,10] \to \mathbb{R}$ is defined as $f(x) = \begin{cases} 0, & 0 \le x \le 2 \\ 1, & 2 \le x \le 5 \\ 0 & 5 < x \le 10 \end{cases}$ and $F(x) = \int_0^x f(t)\,dt$ then

(a) $F(x) = 3$ for $x \le 5$ (b) $F'(x) = f(x)$ for every x

(c) F is not differentiable at $x = 2$ and $x = 5$ (d) F is differentiable everywhere on [0, 10]

18. The Maclaurin series expansion $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots\ldots$ is valid

(a) Only if $x \in [-1,1]$ (b) if $x > -1$

(c) only if $x \in (-1,1]$ (d) for every $x \in \mathbb{R}$

19. If $4x \equiv 2 (\mathrm{mod}\, 6)$ and $3x \equiv 5 (\mathrm{mod}\, 8)$, then one of the value of x is

(a) 32 (b) 34

(c) 26 (d) 23

20. If $f(x) = \lim_{x \to \infty} S_n(x)$, where $S_n(x) = \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \ldots + \frac{x}{(nx+1)((n+1)x+1)}$, then the function f is

(a) continuous nowhere

(b) continuous everywhere

(c) continuous everywhere except at countably many points

(d) continuous everywhere except at one point

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21. The rate of change of $f(x, y) = 4y - x^2$ at the ponit (1, 5) in the direction from (1, 5) to the point (4, 3) is

(a) $\frac{-6}{\sqrt{5}}$ (b) $\frac{-14}{\sqrt{13}}$

(c) $\frac{-12}{\sqrt{5}}$ (d) $\frac{-19}{\sqrt{13}}$

22. Let $G = \{a_1, a_2, ..., a_{25}\}$ be a group of order 25. For $b, c \in G$ let

$$bG = \{ba_1, ba_2, ...ba_{25}\}, Gc = \{a_1c, a_2c, ..., a_{25}c\}.$$

Then

(a) $bG = Gc$ only if $b = c$ (b) $bG = Gc \forall b, c \in G$

(c) $bG = Gc$ only if $b^{-1} = c$ (d) $bG \neq Gc$, if $b \neq c$

23. If $\langle x_n \rangle$ is a sequence such that $x_n \geq 0$, for every $n \in \mathbb{N}$ and if $\lim_{x\to\infty}((-1)^n x_n)$ exists then which one of the following statements is true ?

(a) The sequence $\langle x_n \rangle$ is a Cauchy sequence (b) The sequence $\langle x_n \rangle$ is not a Cauchy sequence

(c) The sequence $\langle x_n \rangle$ is unbounded. (d) The sequence $\langle x_n \rangle$ is divergent

24. If $n > 2$, then $n^5 - 5n^3 + 4n$ is divisble by

(a) 80 (b) 120

(c) 100 (d) 125

25. Let $S = \bigcap_{n=1}^{\infty}\left[2 - \frac{1}{n}, 3 + \frac{1}{n}\right]$. The S equals

(a) (2, 3] (b) [2, 3]

(c) [2, 3) (d) (2, 3)

26. If $a_n = n^{\sin\left(\frac{n\pi}{2}\right)}$, then

(a) $\limsup a_n = +\infty, \liminf a_n = -1$ (b) $\limsup a_n = +\infty, \liminf a_n = 0$

(c) $\limsup a_n = +\infty, \liminf a_n = -\infty$ (d) $\limsup a_n = 1, \liminf a_n = 0$

27. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as $f(x, y) = |x| + |y|$. Then which one of the following statements is true?

(a) f is continuous at (0, 0) and $f_x(0,0) \neq f_y(0,0)$

(b) f is continuous at (0, 0) and $f_x(0,0) = f_y(0,0)$

(c) f is discontinuous at (0, 0) and $f_x(0,0) = f_y(0,0)$

(d) f is continuous at (0, 0) but f_x and f_y does not exist at (0, 0)

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28. Let A and B be two subsets of a metric space X. If int$A$ denotes the interior of A then which one of the following statements is not true ?

(a) $A \subseteq B \Rightarrow \text{int}\, A \subseteq \text{int}\, B$ (b) $\text{int}(A \cup B) = \text{int}\, A \cup \text{int}\, B$

(c) $\text{int}(A \cap B) = \text{int}\, A \cap \text{int}\, B$ (d) $\text{int}(A \cup B) \supset \text{int}\, A \cup \text{int}\, B$

29. Which one of the following statements is false ?

(a) A subring of a field is a subfield

(b) A subring of the ring of integers $\mathbb{Z}$, is an ideal of $\mathbb{Z}$

(c) A commutative ring with unity is a field if it has no proper ideals

(d) A field has no proper ideals

30. Let $\sigma = (37125)(43216) \in S_7$, the symmetric group of degree 7. The order σ is

(a) 7 (b) 4

(c) 5 (d) 2

31. Let $S = \bigcap_{n=1}^{\infty} \left[0, \frac{1}{n}\right]$. Then which one of the following statements is true ?

(a) $\inf S > 0$ (b) $\sup S = 1$ and $\inf S = 0$

(c) $\sup S > 0$ (d) $\sup S = \inf S = 0$

32. The characteristics of the partial differential equation $36\frac{\partial^2 z}{\partial x^2} - y^{14}\frac{\partial^2 z}{\partial y^2} - 8x^{12}\frac{\partial z}{\partial x} = 0$ when it is of hyperbolic type are given by

(a) $x + \frac{36}{y^6} = C_1, x - \frac{36}{y^6} = C_2$ (b) $x + \frac{1}{y^6} = C_1, x - \frac{1}{y^6} = C_2$

(c) $x + \frac{1}{y^7} = C_1, x - \frac{1}{y^7} = C_2$ (d) $x + \frac{36}{y^7} = C_1, x - \frac{36}{y^7} = C_2$

33. A bound for the error for the trapezoidal rule for the definite integral $\int_0^1 \frac{1}{1+x}dx$ is

(a) $\frac{1}{6}$ (b) $\frac{2}{25}$

(c) $\frac{1}{15}$ (d) $\frac{1}{20}$

34. Exact value of the definite integral $\int_a^b f(x)\,dx$ using Simpson's rule

(a) cannot be given for any polynomial (b) is given when $f(x)$ is a polynomial of degree 4

(c) is given when $f(x)$ is a polynomial of degree 5 (d) is given when $f(x)$ is a polynomial of degree 2

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35. Let p be a prime and let G be a non-abelian p-group. The least value of m such that $p^m \setminus o\left(\frac{G}{z(G)}\right)$ is

(a) 0 (b) 1

(c) 3 (d) 2

36. If φ is Euler's Phi function then the value of $\varphi(720)$ is

(a) 248 (b) 144

(c) 192 (d) 72

37. The total number of arithmetic operations required to find the solution of a system of n linear equations in n unknowns by Gauss elimination method is

(a) $\frac{2}{3}n^3+\frac{1}{2}n^2-\frac{5}{6}n$ (b) $n^3-\frac{1}{6}n$

(c) $\frac{2}{3}n^3+\frac{3}{2}n^2-\frac{7}{6}n$ (d) $\frac{1}{3}n^3+\frac{1}{2}n^2-\frac{5}{6}n$

38. If $\langle x_n \rangle$ is a sequence defined as $x_n=\left[\frac{5+n}{2n}\right]$, for every $n\in\mathbb{N}$ where [.] denotes the greatest integer function then $\lim_{x\to\infty} x_n$

(a) 1 (b) $\frac{1}{2}$

(c) does not exist (d) 0

39. Let R be a ring with characteristic n where $n \geq 2$.The M is the ring of 2×2 matrices over R then the characteristic of M is

(a) 1 (b) 0

(c) n - 1 (d) n

40. If $A=\begin{bmatrix} a & 2 \\ 1 & b \end{bmatrix}$ is a matrix eigen values $\sqrt{6}$ and $-\sqrt{6}$ then the values of a and b are respectively.

(a) 2 and -1 (b) 2 and -2

(c) 2 and 1 (d) -2 and 1

41. The dimension of the vector space of all 6×6 real skew-symmetric matrices is

(a) 36 (b) 21

(c) 30 (d) 15

42. Let $(x_0, f(x_0))=(0,-1),(x_1, f(x_1))=(1,a)$ and $(x_2, f(x_2))=(2,b)$. If the first order divided differences $f[x_0,x_1]=5$ and $f[x_1,x_2]=c$ and the second order divided difference $f[x_0,x_1,x_2]=-\frac{3}{2}$, then the values of a, b and c are

(a) 4, 2, 4 (b) 2, 4, 6

(c) 4, 6, 2 (d) 6, 2, 4

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43. Let the polynomial $f(x) = 3x^5 + 15x^4 - 20x^3 + 10x + 20 \in \mathbb{Z}[x]$ and $f_0(x)$ be polynomial in $\mathbb{Z}_3[x]$ obtained by reducing the coefficients of $f(x)$ modulo 3. Which one of the following statements is true ?

(a) $f(x)$ is reducible over $\mathbb{Q}$, $f_0(x)$ is reducible over $\mathbb{Z}_3$

(b) $f(x)$ is irreducible over $\mathbb{Q}$, $f_0(x)$ is reducible over $\mathbb{Z}_3$

(c) $f(x)$ is reducibe over $\mathbb{Q}$, $f_0(x)$ is irreducible over $\mathbb{Z}_3$

(d) $f(x)$ is ireducible over $\mathbb{Q}$, $f_0(x)$ is irreducible over $\mathbb{Z}_3$

44. The general solution of the system of the differential equations

$$x_1' = 3x_1 - 2x_2$$

$$x_2' = 2x_1 - 2x_2$$

is given by

(a) $\begin{pmatrix} c_1e^{-t} + 2c_2e^{2t} \\ 2c_1e^{-t} + c_2e^{2t} \end{pmatrix}$ (b) $\begin{pmatrix} c_1e^{t} + 2c_2e^{-2t} \\ 2c_1e^{t} + 2c_2e^{-2t} \end{pmatrix}$

(c) $\begin{pmatrix} c_1e^{t} + 2c_2e^{-2t} \\ c_1e^{t} + c_2e^{-2t} \end{pmatrix}$ (d) $\begin{pmatrix} c_1e^{-t} + 2c_2e^{2t} \\ c_1e^{-t} - c_2e^{2t} \end{pmatrix}$

45. The eigenvalues for the Sturm-Liouville problem

$$y'' + \lambda y = 0, 0 \le x \le \pi,$$

$$y(0) = 0, y'(\pi) = 0$$

are

(a) $\lambda_n = n^2\pi^2, n = 1, 2, ...$ (b) $\lambda_n = n^2, n = 1, 2, ...$

(c) $\lambda_n = n\pi, n = 1, 2, ...$ (d) $\lambda_n = \frac{(2n-1)^2}{4}, n = 1, 2, ...$

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46. The initial value problem

$$x\frac{dy}{dx} - 2y = 0$$

$$x > 0, y(0) = 0$$

has

(a) exactly two solutions (b) a unique solution

(c) no solution (d) infinitely many solutions

47. The partial differential equation $(x^2 - 1)\frac{\partial^2 z}{\partial x^2} + 2y\frac{\partial^2 z}{\partial x \partial y} - \frac{\partial^2 z}{\partial y^2} = 0$ is

(a) hyperbolic for $\{(x, y) \in \mathbb{R} : x^2 + y^2 < 1\}$ (b) parabolic for $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$

(c) hyperbolic for $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 1\}$ (d) elliptic for $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 1\}$

48. Let f be a convex function with $f(0) = 0$. Then the function g defined on $(0, +\infty)$ as $g(x) = \frac{f(x)}{x}$

(a) is an incresing function

(b) is such that its monotonicity cannot be determined

(c) is nether increasing nor decreasing function

(d) is a decreasing function

49. Which one of the statements is false ?

(a) Every quotient group of a cyclic group is cyclic

(b) If G and H are groups and $f : G \to H$ is a honomorphism then f induces an isomorphism of $\frac{G}{\text{Ker}(f)}$ with H

(c) Every quotient group of an abelian group is abelian

(d) If G is a group and $Z(G)$ is its centre such that the quotient group of G by $Z(G)$ is cyclic, then G is abelian

50. For cubic spline interpolation which one of the following statements is true ?

(a) The second derivatives of the splines are continuous at the interior data points but not the first derivatives

(b) The third derivatives of the splines are continuous at the interior data points

(c) The first derivatives of the splines are continuous at the interior data points but not the second derivatives

(d) The first and the second derivatives of the splines are continuous at the interior data points

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ANSWER KEY

1. (a)	2. (d)	3. (c)	4. (a)	5. (d)
6. (a)	7. (b)	8. (d)	9. (a)	10. (c)
11. (c)	12. (b)	13. (d)	14. (c)	15. (c)
16. (d)	17. (c)	18. (c)	19. (d)	20. (d)
21. (b)	22. (b)	23. (b)	24. (b)	25. (b)
26. (b)	27. (b)	28. (d)	29. (a)	30. (b)
31. (d)	32. (b)	33. (a)	34. (d)	35. (d)
36. (c)	37. (c)	38. (d)	39. (d)	40. (b)
41. (b)	42. (c)	43. (b)	44. (a)	45. (d)
46. (d)	47. (c)	48. (a)	49. (b)	50. (d)

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