HYDERABAD CENTRAL UNIVERSITY (HCU)
M.Sc. Mathematics Entrance - 2013

Time : 2 Hours **Max. Marks: 100**

<u>Instructions</u>:

(*i*) There are a total of **50** questions in **Part-A** and **Part-B** together.

(*ii*) There is a negative marking in **Part-A**. Each correct answer carries **1 mark** and each wrong answer carries **–0.33 mark**. Each question in **Part-A** has only one correct option.

(*iii*) There is **no negative** marking in **Part-B**. Each correct answer carries **3 marks**. In **Part-B** some questions have **more than** one correct option. All the correct options have to be marked in OMR sheet other wise **zero marks** will be credited.

PART-A

1. We say that a sequence (a_n) does NOT converge to l if

(a) $\forall \in > 0, \forall n_0 \in \mathbb{N}, \forall n \ge n_0$ we have $|a_n - l| > \in$

(b) $\forall \in > 0, \forall n_0 \in \mathbb{N}, \exists n \ge n_0$ such that $|a_n - l| > \in$

(c) $\exists \in > 0, \forall n_0 \in \mathbb{N}, \exists n \ge n_0$ such that $|a_n - l| > \in$

(d) $\exists \in > 0, \forall n_0 \in \mathbb{N}, \forall n \ge n_0$ we have $|a_n - l| > \in$

2. Consider a sequence (a_n) of positive numbers satisfying the condition $a_n a_{n+2} \le a_{n+1}^2, \forall n \in \mathbb{N}$ then (a_n) is a

(a) convergent sequence if $a_1 \ne 2a_2$

(b) monotonically increasing sequence if $a_1 \ne 2a_2$

(c) convergent sequence if $a_1 = 2a_2$

(d) monotonically increasing sequence if $a_1 = 2a_2$

3. The sum of the series $\sum_{n=1}^{\infty}[(n+1)^{\frac{1}{5}} - n^{\frac{1}{5}}]$ is

(a) less than –1
(b) equal to –1
(c) greater than –1 but less than 2
(d) none of the above

4. Let $S=\{x\in\mathbb{R}/x^2\le 5\}\cap\mathbb{Q}$. Which of the following statements is true about S'?

(a) S is bounded above the $\sup S\in\mathbb{Q}$

(b) S is bounded above and $\sup S\in\mathbb{R}-\mathbb{Q}$

(c) S is a closed interval

(d) S is an open interval

5. The value of $\lim\limits_{x\to 0}\dfrac{e^{(1/x)}-e^{(-1/x)}}{e^{(1/x)}+e^{(-1/x)}}$ is

(a) 0 (b) 1 (c) –1 (d) none of the above

6. Consider the function $f:\mathbb{R}\to\mathbb{R}$ given by $f(x)=\begin{cases}-x+3, & x\in\mathbb{Q},\\ x^2-6x+9 & x\notin\mathbb{Q}\end{cases}$. The set of all points at which f is continuous is

(a) $\{2, 3\}$ (b) $\{3\}$ (c) $\mathbb{R}-\{2,3\}$ (d) $\mathbb{R}-\{3\}$

7. Consider the function $f:\mathbb{R}\to\mathbb{R}$ given by $f(x)=\begin{cases}\sin x, & x\ge 0,\\ 1-\cos x, & x<0\end{cases}$. Which of the following statements is true about f?

(a) f is differentiable (b) f is continuous but NOT differentiable

(c) f is discontinuous (d) none of the above statements is true

8. Let $f:[0,1]\to\mathbb{R}, g:[0,1]\to\mathbb{R}$ given by $f(x)=\begin{cases}1, & x\in\mathbb{Q}\cap[0,1]\\ 0, & x\notin\mathbb{Q}\cap[0,1]\end{cases}$ and $g(x)=\begin{cases}0, & x\in\mathbb{Q}\cap[0,1]\\ 1, & x\notin\mathbb{Q}\cap[0,1]\end{cases}$, then

(a) both f and g are Riemann integrable

(b) f is Riemann integrable but g is NOT Riemann integrable

(c) g is Riemann integrable but f is NOT Riemann integrable

(d) both f and g are NOT Riemann integrable

9. $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\dfrac{2k}{k^2+n^2}=$

(a) 0 (b) log 2 (c) 2 (d) ∞

10. A solution of $xdy-ydx+(x^2+y^2)dx+(x^2+y^2)dy=0$ is

(a) $\arctan(y/x)+x+y=C$ (b) $\dfrac{y}{x}+x^2+y^2=C$

(c) $\arctan(y/x)+x^2+y^2=C$ (d) $\dfrac{y}{x}+x+y=C$

11. The general solution of $(D^4+I)^2 y = 0$ is

(a) $C_1 \sin x + C_2 \cos x + C_3 e^x + C_4 e^{-x}$

(b) $C_1 x \sin x + C_2 x \cos x + C_3 e^x + C_4 e^{-x}$

(c) $(C_1 + C_2 x)\sin x + (C_3 + C_4 x)\cos x + C_5 e^x + C_6 e^{-x}$

(d) $(C_1 + C_2 x)\sin x + (C_3 + C_4 x)\cos x + (C_5 + C_6 x)e^x + (C_7 + C_8 x)e^{-x}$

12. Consider three different planes $a_{11}x + a_{12}y + a_{13}z = d_1, a_{21}x + a_{22}y + a_{23}z = d_2$ and $a_{31}x + a_{32}y + a_{33}z = d_3$. Let $A = (a_{ij}),\ 1 \le i, j \le 3$. Which of the following conditions necessarily implies that there exists a unique point of intersection of all three planes?

(a) $\det(A) = 0$
(b) $\det(A) \ne 0$
(c) $\text{Trace}(A) = 0$
(d) $\text{Trace}(A) \ne 0$

13. The number of planes containing both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x-2}{-1} = \frac{y-4}{-5} = \frac{z-6}{-1}$ is

(a) 0
(b) 1
(c) more than 1 but finitely many
(d) infinite

14. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 22 \\ 0 & 1/7 & \pi \end{bmatrix}$, then $\det(A)$ is

(a) zero
(b) a non-zero rational number
(c) an irrational number less than 1
(d) an irrational number greater than 1

15. Consider the vector space $\mathbb{R}^3$ over $\mathbb{R}$ and $A, B \subset \mathbb{R}^3$ such that $0 \notin A \cup B$. Let the number of elements in A and B are 4 and 2 respectively, then

(a) both A and B are linearly dependent sets
(b) A is linearly dependent set but B is linearly independent set
(c) both A and B are linearly independent sets
(d) none of the above is a true statement

16. The number of group homomorphisms from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{13}$ is

(a) 0
(b) 1
(c) more than 1 but finitely many
(d) infinite

17. The centre of $\mathbb{Z}_{33}$ is

(a) $\{0\}$
(b) $\mathbb{Z}_3$
(c) $\mathbb{Z}_{11}$
(d) $\mathbb{Z}_{33}$

18. Let G be a group and H be a subgroup of G. Which of the following statements is true?

(a) If H is a normal subgroup of G then $gH = Hg, \forall g \in G$

(b) If H is a normal subgroup of G then $gH \neq Hg,$ for some $g \in G$

(c) If $gH = Hg,$ for some $g \in G$ then H is a normal subgroup of G

(d) If $gH \neq Hg,$ for some $g \in G$ then H is a normal subgroup of G

19. The number of elements of order 8 in a cyclic group of order 16 is

(a) 1 (b) 2 (c) 3 (d) 4

20. If $x \neq e, y \neq e$ are elements in a group G such that the order of x is 2 and $x^{-1}yx = y^2$ then the order of y is

(a) 1 (b) 2 (c) 3 (d) 4

21. In the ring $(\mathbb{Z}, +, .)$ the set $\{12u + 30v \mid u, v \in \mathbb{Z}\}$ is the same as $n\mathbb{Z}$ for $n =$

(a) 6 (b) 4 (c) 3 (d) 2

22. Let S be the sphere with center at the origin and radius 1. Let $\overline{f}$ is a vector field given by

$\overline{f}(x, y, z) = (z - 2xyz)\hat{i} + 9x^2yz^2\hat{j} + (yz^2 - 3x^2z^3)\hat{k}$. If $\hat{n}$ is the outward normal then, the value of

$\iint_S \overline{f} \cdot \hat{n} dS =$

(a) 0 (b) $\frac{4}{3}\pi$ (c) π (d) $\frac{4}{3}\pi^3$

23. If ϕ is a real valued smooth function and $\overline{f}$ is a vector valued smooth function on $\mathbb{R}^3$. then $\text{div}(\phi \, \text{Curl} \, \overline{f}) =$

(a) $\nabla\phi \cdot \text{Curl} \, \overline{f}$ (b) $\nabla(\overline{f} \cdot \nabla\phi)$

(c) $\nabla\phi \cdot \text{Curl} \, \overline{f} + \nabla(\overline{f} \cdot \nabla\phi)$ (d) none of the above

24. What is the probability of that girls out number boys in a family with 5 children. Assume that births are independent trials and probability of a boy is equal to 1/2.

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{15}{32}$ (d) $\frac{17}{32}$

25. Consider two boxes numbered Box 1 and Box 2. Let Box 1 contains 5 red balls and 4 black balls Box 2 contains 10 red balls and 17 black balls. Consider a random experiment of choosing a box, picking a ball from it. What is the probability that the color of the ball is red?

(a) $\frac{25}{54}$ (b) $\frac{50}{54}$ (c) $\frac{15}{36}$ (d) $\frac{15}{17}$

PART-B

26. Consider the statement "There is a train in which every compartment has at least one passenger without the ticket". Negation of this statement is

(a) There is a train in which every compartment has at least one passenger with the ticket

(b) There is a train in which every passenger of every compartment has the ticket

(c) Every train has a compartment in which every passenger has the ticket

(d) In every train every passenger in every compartment has the ticket

27. Consider a sequence (a_n) of real numbers. Which of the following conditions imply that (a_n) is convergent?

(a) $|a_{n+1} - a_n| < \frac{1}{n}, \forall n \in \mathbb{N}$

(b) $|a_{n+1} - a_n| < \frac{1}{3^n}, \forall n \in \mathbb{N}$

(c) $a_n > 0, \forall n \in \mathbb{N}$ and a_n is monotonically increasing

(d) $a_n > 0, \forall n \in \mathbb{N}$ and a_n is monotonically decreasing

28. Which of the following series are convergent?

(a) $\sum_{n=0}^{\infty} \frac{\log n}{n^{3/2}}$ (b) $\sum_{n=0}^{\infty} \frac{n^2}{n!}$ (c) $\sum_{n=0}^{\infty} \frac{1}{n \log n}$ (d) $\sum_{n=0}^{\infty} \frac{e^n}{n^{100}}$

29. Which of the following statements are true?

(a) If $A \subset \mathbb{Q}$ such that $\mathbb{Q} - A$ is finite then A is dense in $\mathbb{R}$

(b) There exists $A \subset \mathbb{Q}$ such that $\mathbb{Q} - A$ is finite and A is dense in $\mathbb{R}$

(c) There exists a pair of disjoint subsets of Q such that both of them are dense in $\mathbb{R}$

(d) None of the above is a true statement

30. Consider $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = \sin^3(|x|)$, then $f'(0)$

(a) is equal to –1 (b) is equal to 0

(c) is equal to 1 (d) does not exist

31. Consider the following two statements

S_1 : If $f : [0,1] \to [0,1]$ is continuous then $\exists x_0 \in [0,1]$ such that $f(x_0) = x_0$

S_2 : There exists a continuous function $f : [0,1] \to [0,1] - \left\{\frac{1}{2}\right\}$ such that *f* is on to

(a) Both S_1 and S_2 are true (b) S_1 is true but S_2 is false

(c) S_2 is true but S_1 is false (d) Both S_1 and S_2 are false

32. Consider the following two statements.

$$S_1 : \int_0^{\pi/2} \frac{\sin x}{x} dx \text{ exists}$$

$$S_2 : \int_0^1 \frac{x}{\log x} dx \text{ exists}$$

(a) Both S_1 and S_2 are true
(b) S_1 is true but S_2 is false
(c) S_2 is true but S_1 is false
(d) Both S_1 and S_2 are false

33. Solution of $(x^2+y^2)xdx+(x^2+y^2)ydy+2xy(xdy-ydx)=0$ is

(a) $\log(\sqrt{x^2+y^2})-\dfrac{x^2}{x^2+y^2}=C$
(b) $\log(x^2+y^2)-\dfrac{x^2}{x^2+y^2}=C$
(c) $\log(\sqrt{x^2+y^2})-\tan^{-1}\dfrac{y}{x}=C$
(d) $\log(x^2+y^2)-\tan^{-1}\dfrac{y}{x}=C$

34. The general solution of $(D^2-1)y=x^2+e^{-x}$ is

(a) $C_1e^x+C_2e^{-x}-\left[\frac{1}{4}(2x+1)e^{-x}+x^2+2\right]$
(b) $C_1\sin x+C_2\cos x-\left[\frac{1}{4}(2x+1)e^{-x}+x^2+2\right]$
(c) $C_1e^x+C_2e^{-x}-\left[\frac{1}{2}e^{-x}+x^2+2\right]$
(d) $C_1\sin x+C_2\cos x-\left[\frac{1}{2}e^{-x}+x^2+2\right]$

35. The value of k such that the lines $\dfrac{x-1}{k}=\dfrac{y-1}{4}=\dfrac{z-2}{3}$ and $\dfrac{x+1}{2}=\dfrac{y-1}{1}=\dfrac{z}{3}$ are coplanar is

(a) –1
(b) 1
(c) –2
(d) 2

36. Consider a plane which is at a distance p from the origin O = (0, 0, 0). Let A, B, C be the points of intersection of that plane with the co-ordinate axis. The locus of the centre of the sphere passing through O, A, B and C is

(a) $\dfrac{2}{x^2}+\dfrac{2}{y^2}+\dfrac{2}{z^2}=\dfrac{1}{p^2}$
(b) $\dfrac{1}{x^2}+\dfrac{1}{y^2}+\dfrac{1}{z^2}=\dfrac{4}{p^2}$
(c) $\dfrac{1}{x^2}+\dfrac{1}{y^2}+\dfrac{1}{z^2}=\dfrac{2}{p^2}$
(d) $\dfrac{4}{x^2}+\dfrac{4}{y^2}+\dfrac{4}{z^2}=\dfrac{1}{p^2}$

37. Consider the circle C which is the intersection of the sphere $x^2+y^2+z^2-x-y-z=0$ and the plane $x+y+z=1$. The radius of the sphere with centre at the origin, containing the circle C is

(a) 1
(b) 2
(c) 3
(d) 4

38. Which of the following statements are true?

(a) All groups of order 4 are abelian
(b) All groups of order 6 are abelian
(c) $73^{12}-1$ is divisible by 7
(d) A subgroup of a cyclic group must be cyclic

39. Consider the quotient group $G = \frac{\mathbb{Q}}{\mathbb{Z}}$ under addition. Which of the following statements about G are true?

(a) G is a finite group
(b) In G every element has a finite order
(c) G has no non-trivial proper subgroups
(d) G is NOT a cyclic group

40. Let $G = \left\{ \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} \middle| a \in \mathbb{Q} - \{0\}, b \in \mathbb{Q} \right\}, U = \left\{ \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \middle| b \in \mathbb{Q} \right\}, \ D = \left\{ \begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix} \middle| a \in \mathbb{Q} - \{0\} \right\}$

Which of the following statements are true?

(a) G, U, D are all groups under multiplication
(b) D is a normal subgroup of G
(c) U is a normal subgroup of G
(d) For every matrix $A \in U$, $ADA^{-1} \subseteq D$

41. Let $X = \{1, 2, 3, 4, 5\}$, $P(X)$ be the power set of X. Consider the ring $R = (P(X), \Delta, \cap)$, for subsets A and B of X, $A \Delta B = (A \cup B) - (A \cap B)$. Which of the following statements are true about R?

(a) R is a commutative ring with unity
(b) R is field
(c) Every element in R has 'additive' order 2.
(d) Every element in R has 'multiplicative' order 2

42. Consider the ring $R = (\mathbb{Z}_{60}, +, .)$. Which of the following statements are true about R?

(a) There are no maximal ideals in R
(b) There are three maximal ideals in R
(c) There are ten non-zero proper ideals in R
(d) All non-zero ideals in R are maximal

43. Consider the group $\mathbb{Z}$ under addition +. Define the binary operation * on $\mathbb{Z}$ by $a * b = 0, \forall a \cdot b \in \mathbb{Z}$. Which of the following statements are true about R?

(a) $(\mathbb{Z}, +, .)$ is a commutative ring with unity
(b) $(\mathbb{Z}, +, .)$ is a ring
(c) Every additive subgroup of $\mathbb{Z}$ is an ideal
(d) The only ideals in $\mathbb{Z}$ are of the form $n\mathbb{Z} = \{nx \mid x \in \mathbb{Z}\}$

44. Let A be a non-singular 3 × 3 matrix with real entries. For every non-zero eigenvalue λ of A,

(a) λ is an eigenvalue of both $P^{-1}AP, PAP^{-1}$ where $\det(P) \neq 0$
(b) $1 + \lambda$ is an eigenvalue of $I + A$
(c) if $\det(A) < 1$ then $|\lambda| < 1$
(d) if μ is an eigen value of A^{-1} then $\mu\lambda = 1$

45. Let A be a 2 × 2 real matrix. Let the sum of the entries in each row of A be qual to 2. Which of the following statements is true?

(a) 0 is always an eigen value of A
(b) 0 and 2 are always eigenvalues of A
(c) 2 is always an eigenvalues of A
(d) None of the above

46. Let A, B be a 4 × 4 matrices. Denote rank of a matrix A, B by $\rho(A), \rho(B)$ and adjoint of A by adj(A). Which of the following statements are true?

(a) $\rho(A+B) \le \rho(A)+\rho(B)$
(b) $\rho(A-B) \le \rho(A)-\rho(B)$
(c) $\rho(AB) \le \rho(A)\rho(B)$
(d) If $\rho(A)=2$ then adj (A) = $O_{4\times4}$

47. Consider the vector space $V=\mathbb{R}^3(\mathbb{R})$ and $B=\{v_1, v_2\}\subset V, 0\notin B$. Which of the following statements are true?

(a) If B is a linearly dependent set then $\exists(\alpha_1,\alpha_2)\ne(0,0)$ such that $\alpha_1 v_1+\alpha_2 v_2=0$

(b) If B is a linearly dependent set then $\exists(\alpha_1\cdot\alpha_2)$ such that $\alpha_1\ne 0,\ \alpha_2\ne 0$ and $\alpha_1 v_1+\alpha_2 v_2=0$

(c) If B is linearly independent then $\exists$ no nonzero 2-tuple (α_1,α_2) such that $\alpha_1 v_1+\alpha_2 v_2\ne 0$

(d) If B is linearly independent then $\exists$ no nonzero 2-tuple (α_1,α_2) such that $\alpha_1 v_1+\alpha_2 v_2=0$

48. Let $\overline{r}=x\hat{i}+y\hat{j}+z\hat{k},\ |\overline{r}|=\sqrt{x^2+y^2+z^2}$ and $\overline{f}:\mathbb{R}^3-\{0\}\to\mathbb{R}^3$ be given by $f(x,y,z)=\dfrac{\overline{r}}{|\overline{r}|^n}$. The value of n for which $div(\overline{f})=0$ is

(a) 1
(b) 2
(c) 3
(d) 4

49. Let R be a region in the *xy*-plane. The boundary of R is a smooth simple closed curved C which is parametrized by $C=(x(t), y(t)), t\in[0,1]$. The area of R is NOT equal to

(a) $\int_0^1 x(t)y'(t)dt$
(b) $-\int_0^1 y(t)x'(t)dt$
(c) $\frac{1}{2}\int_0^1 (x(t)y'(t)+y(t)x'(t))dt$
(d) $\frac{3}{4}\int_0^1 x(t)y'(t)dt-\frac{1}{4}\int_0^1 y(t)x'(t)dt$

50. A storage depot contains 10 machines 4 of which are defective. If a company selects 5 of these machines randomly, then what is the probability that at least 4 of the machines are NON DEFECTIVE?

(a) $\frac{11}{42}$
(b) $\frac{5}{21}$
(c) $\frac{1}{252}$
(d) None of the above

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ANSWER KEY

PART-A

1.	(c)	**2.**	(c)	**3.**	(d)	**4.**	(b)	**5.**	(d)
6.	(a)	**7.**	(b)	**8.**	(d)	**9.**	(b)	**10.**	(a)
11.	(d)	**12.**	(b)	**13.**	(__)	**14.**	(c)	**15.**	(d)
16.	(b)	**17.**	(d)	**18.**	(a)	**19.**	(d)	**20.**	(c)
21.	(a)	**22.**	(a)	**23.**	(a)	**24.**	(b)	**25.**	(a)

PART-B

26.	(c)	**27.**	(b, d)	**28.**	(a, b)	**29.**	(a, b, c)	**30.**	(b)
31.	(b)	**32.**	(a)	**33.**	(__)	**34.**	(a)	**35.**	(__)
36.	(__)	**37.**	(__)	**38.**	(a, c, d)	**39.**	(__)	**40.**	(__)
41.	(a, c)	**42.**	(b, c)	**43.**	(__)	**44.**	(a, b, d)	**45.**	(c)
46.	(a, c, d)	**47.**	(a, b, d)	**48.**	(c)	**49.**	(b, d)	**50.**	(a)