HYDERABAD CENTRAL UNIVERSITY (HCU)
M.Sc. Mathematics Entrance - 2018

Time : 2 Hours **Max. Marks: 100**

Instructions:

(*i*) There are a total of **50** questions in **Part-A** and **Part-B** together.

(*ii*) There is a negative marking in **Part-A**. Each correct answer carries **1 mark** and each wrong answer carries **–0.33 mark**. Each question in **Part-A** has only one correct option.

(*iii*) There is **no negative** marking in **Part-B**. Each correct answer carries **3 marks**. In **Part-B** some questions have **more than** one correct option. All the correct options have to be marked in OMR sheet other wise **zero marks** will be credited.

PART - A

1. Let $X \subset \mathbb{N}$ be a nonempty finite set and $Y \subset \mathbb{N}$ be an infinite set. Define $A = \{x - y : x \in X \text{ and } y \in Y\}$. then

   (a) $\inf(A) = -\infty$ and $\sup(A) = \infty$
   (b) $\inf(A) = -\infty$ and $\sup(A) < \infty$
   (c) $\inf(A) > -\infty$ and $\sup(A) = \infty$
   (d) $\inf(A) > -\infty$ and $\sup(A) < \infty$

2. Let X be a nonempty set and $A, B \subset X$. If $(A \cup B) \setminus (A \cap B)$ is a finite set, then

   (a) Both A and B are finite sets.
   (b) Atleast one of A, B is a finite set.
   (c) X is a finite set.
   (d) None of the above

3. Let $f, g : \mathbb{R} \to \mathbb{R}$ be a polyromials with $\deg(f) = 5$ and $\deg(g) = 7$. Let $A = \{x \in \mathbb{R} : f(x) = g(x)\}$. Then

   (a) $|A| \le 5$
   (b) $|A| \le 7$
   (c) A is finite, but $|A|$ can be arbitrarily large
   (d) A can be finite

4. Let X be a set with $2m + 1$ elements, where $m \ge 4$. The number of subsets of X having at least $m + 1$ elements is

   (a) 2^{2m+1}
   (b) 2^{2m-1}
   (c) 2^{2m}
   (d) $2^m + 1$

5. A box contain RED, BLUE and GREEN coloured balls. If all but 18 of the balls are BLUE, all but 12 of the balls are GREEN and all but 24 balls are RED, then which of the following is true

   (a) Box contains more RED balls than BLUE or GREEN ones.
   (b) Box contains more BLUE balls than RED or GREEN ones.

(c) Box contains more GREEN balls than RED or BLUE ones.

(d) none of the above.

6. Vinod and Binod are two students among a group of n students. The number of ways to allocate n different rooms in a line to this group of students so that Vinod and Binod are not in adjacent rooms is

(a) $(n-2)(n-1)!$ (b) $2(n-1)!$

(c) $(n-1)!$ (d) $2(n-2)!$

7. Let G be a group and $a,b \in G$ such that $o(a)=6, o(b)=2$ and $a^3b = ba$. Then $o(ab)$ is

(a) 6 (b) 8

(c) 12 (d) 2

8. Suppose G is a group and $x \in G$ such that order of x is $\geq |G|//2$. Then

(a) G is cyclic group (b) If G is abelian, then G is cyclic

(c) If G is finite, then G is cyclic (d) none of the above

9. Let $A\Delta B$ denote the symmetric difference of two sets : $A\Delta B := (A \setminus B) \cup (B \setminus A)$. Which of the following is true ?

(a) $A \cap (B\Delta A) = \phi$ always holds true.

(b) $(A \cup (A\Delta B)) \cap B \neq B$ always holds true.

(c) $A\Delta B = (A \cup B) \cap (A^\circ \cap B^\circ)$ always holds true.

(d) None of the above is true.

10. Let A be a 5×5 real matrix. Suppose 0 is one of eigenvalues of A. which of the following statement is true?

(a) System $AX = 0$ has unique solution (b) System $AX = C$ has unique solution for any C

(c) $AX = 0$ has a non-trivial solution (d) none of the above

11. The order of (123)(145) in the permutation group S_5 is

(a) 6 (b) 3

(c) 5 (d) 9

12. The number of subgroups of order 2 in the permutation group S_3 is

(a) 1 (b) 3

(c) 2 (d) 12

13. Which one the following subsets is a subspace of the vector space $\mathbb{R}^3$ over the field $\mathbb{R}$?

(a) $\{(u,v,w) \in \mathbb{R}^3 \mid 2u+3v+4w=0\}$ (b) $\{(u,v,w) \in \mathbb{R}^3 \mid 2u+3v+4w=1\}$

(c) $\{(u,v,w) \in \mathbb{R}^3 \mid u>0, v>0, w<0\}$ (d) $\{(u,v,w) \in \mathbb{R}^3 \mid u,v,w \text{ are rational}\}$

14. The dimension of the vector space $\mathbb{Q}[\sqrt{2}]$ over the field $\mathbb{Q}$ is

(a) 1 (b) ∞

(c) 4 (d) 2

15. Let $V = \{(x_1, x_2, x_3, x_4) \in \mathbb{R} \mid 2x_1 + \pi x_2 = 0, x_2 - x_3 - x_4 = 0\}$. A basis for V

(a) $\{(1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1)\}$

(b) $\left\{\left(1,-\frac{2}{\pi},0,-\frac{2}{\pi}\right),(0,0,1,-1)\right\}$

(c) $\{(\pi,-2,0,-2),(2\pi,-4,0,-4)\}$

(d) $\left\{(0,0,1,-1),\left(\frac{1}{2},-\frac{1}{\pi},0,0\right)\right\}$

16. In a survey, it is found that 80 students know English, 60 know French, 50 know German, 30 know English and French, 20 know French, 20 know French and German, 15 know English and German and 10 students know all the three languages. How many students know English only?

(a) 45
(b) 55
(c) 20
(d) 80

17. If one of the roots of the quadratic equaiton $10x^2 - 29x + (k+4) = 0$ is the reciprocal of the other root, then the value of k is

(a) 4
(b) 29
(c) 6
(d) –3

18. The number of subsets from 100 distinct objects containing an odd number of objects is

(a) $2^{100} - 1$
(b) 2^{99}
(c) 2^{50}
(d) $2^{50} - 1$

19. The proportions of those arrangements of the numerals 1, 2, ... , 9 in which all the multiples of 3 appears consecutively is

(a) $\frac{1}{2}$
(b) $\frac{1}{6}$
(c) $\frac{1}{12}$
(d) $\frac{1}{36}$

20. The rate of convergence of Newton-Raphson method is

(a) 2
(b) 3
(c) 1
(d) 1.618

21. The Laplace transform of $t^{1/2}$ is

(a) $\sqrt{\pi}/2s^{3/2}$
(b) $\sqrt{\pi}/s^{3/2}$
(c) $\sqrt{\pi}/2s^{1/2}$
(d) $\sqrt{\pi}/s^{1/2}$

22. Let f and g be differentiable functions such that $f'(x) = 2g(x)$ and $g'(x) = -f(x)$ and let $T(x) = f(x)^2 - g(x)^2$. Then $T'(x)$ is equal to

(a) $T(x)$
(b) 0
(c) $2f(x)g(x)$
(d) $6f(x)g(x)$

23. If G is an abelian group, then the number of conjugacy classes equal to

(a) $0(G)$
(b) 1
(c) $0(G) - 0(Z(G)$
(d) 2

24. Let p be a prime and $G = \{z \in \mathbb{C}\} \mid z$ is an n^{th}-root of 1, for some $n \in \mathbb{N}\}$. Then,

(a) an infinite abelian group but not cyclic
(b) an infinite cyclic group
(c) it is a finite cyclic group
(d) None of these

25. Let G be a finite group and H is a subrgop of G of index 2. Then

(a) H is normal and $g^2 \in H$ for any $g \in H$
(b) H is normal and $g^2 = e$
(c) H is need not be normal
(d) None of the above

PART - B

26. Let (x_n) be a sequence of positive reals converging to 1. Which of the following sequences converge to 1?

(a) $(\sqrt{x_n})$
(b) (x_n^3)
(c) $(x_n^{1/n})$
(d) $(2^{x_n} - 1)$

27. Let $f : \mathbb{R} \to \mathbb{R}$ and $f(x) = 3x^4 - 2x^5$. Then which of the following are true?

(a) f is surjective
(b) f is injective
(c) f is uniformly continuous on $\mathbb{R}$
(d) f is Lipschitz continuous on [–100, 100]

28. For $k, n \in \mathbb{N}$, Let $T_{k,n}$ be linear map from $\mathbb{R}^k \to \mathbb{R}^n$. Which of the following are true?

(a) $T_{6,7} \circ T_{7,6} : \mathbb{R}^7 \to \mathbb{R}^7$ can never be injective
(b) $T_{6,7} \circ T_{7,6} : \mathbb{R}^7 \to \mathbb{R}^7$ can never be surjective
(c) $T_{7,6} \circ T_{6,7} : \mathbb{R}^7 \to \mathbb{R}^7$ can never be injective
(d) $T_{7,6} \circ T_{6,7} : \mathbb{R}^7 \to \mathbb{R}^7$ can never be surjective

29. The greatest common divisor (gcd) of $5n+3$ and $7n+4$, for all $n \in \mathbb{N}$ is

(a) 1
(b) 5
(c) n
(d) 2

30. How many three digit numbers are divisbile by 6 ?

(a) 142 (b) 150

(c) 148 (d) 166

31. The number of diagonals in an n-gon is

(a) $\frac{n(n-1)}{2}$ (b) $n(n-3)$

(c) $n(n-1)$ (d) $\frac{n(n-3)}{2}$

32. Let $i=\sqrt{-1}$ in $\mathbb{C}$. Then i^i is

(a) 0 (b) 1

(c) $e^{-\frac{\pi}{2}}$ (d) $e^{-i\frac{\pi}{2}}$

33. Let G be an abelian group with the identity e. Which one of the following statements are true.

(a) $H=\{x\in G \mid \text{order of } x \text{ is odd}\}$ is a subgroup of G

(b) $H=\{x\in G \mid \text{order of } x \text{ is even}\}\cup\{e\}$ is a subgroup of G

(c) Every subgroup of G is normal

(d) G is cyclic

34. Let p be a prime number. Consider the group $\mathrm{SL}(2,\mathbb{Z}_p)=\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a,b,c,d\in\mathbb{Z}_p \text{ and } ad-bc=1\right\}$ under the matrix multiplication. Then the order of $A=\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\in \mathrm{SL}(2,\mathbb{Z}_p)$ is

(a) ∞ (b) p

(c) 1 (d) $p-1$

35. In the ring $\mathbb{Z}_8[X]$, the element $4X^2+6X+3$ is

(a) a nilpotent (b) a unit

(c) an idempotent (d) a zero-divisor

36. Suppose that $\phi:\mathbb{Z}_{20}\to\mathbb{Z}_{20}$ is an automorphism such that $\phi(5)=5$. Then the number of possibilities for $\phi(1)$ is

(a) 4 (b) 1

(c) 5 (d) 20

37. Let X and Y be subspaces of finite dimensional vector space V. Let $X + Y = \{x + y \mid x \in X, y \in Y\}$. The dimension of the subspace $X + Y$ is always equal to

(a) $\dim(X+Y) = \dim(X) + \dim(Y) - \dim(X \cap Y)$

(b) $\dim(X+Y) = \dim(X) + \dim(Y)$

(c) $\dim(X+Y) = \max\{\dim(X), \dim(Y)\}$

(d) $\dim(X+Y) = \dim(X) + \dim(Y) + \dim(X \cap Y)$

38. A number is selected from {1, 2, ...100} if every outcome is equally likely, the probability that the selected number is a prime number given that it is odd is

(a) $\frac{1}{4}$ (b) $\frac{12}{25}$

(c) $\frac{1}{3}$ (d) $\frac{14}{25}$

39. The nullity of $n \times n$ matrix all of whose terms are 1 is

(a) 1 (b) n

(c) $n - 1$ (d) $n - 2$

40. The proportion of 2 × 2 nonsingular matrices whose terms are either 0 or 1 is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) $\frac{1}{16}$

41. The value of $\lim_{n\to\infty} \frac{1}{n}\left(1 + \frac{1}{2} + ... + \frac{1}{n}\right)$ is

(a) 0 (b) 1

(c) ∞ (d) e

42. Suppose G is a finite group and H is a subrgoup of G. If $[G : H] = 2$, then which of the following statements are true ?

(a) If $x \in H$ and $y \notin H$, then $xy \in H$ (b) If $x \notin H$ and $y \notin H$, then $xy^{-1} \in H$

(c) If $x \notin H$ and $y \notin H$, $xy \in H$ (d) both (b) and (c) are true

43. Let S_3 be the permutation group on {1, 2, 3}. Then there exists a non-trivial group homomorphism $f : S_3 \to S_3$ such that

(a) Kernel $f = \{(12), e\}$ (b) Kernel $f = \{(123), (132), e\}$

(c) Kernel f can contains (123), (12) (d) None of the above

44. Let $T: V \to W$ be a linear transformation, then

(a) if $v_1, v_2, ...v_n$ are linear independent then $T(v_1), T(v_2)...., T(v_n)$ are linear independent.

(b) if $T(v_1), T(v_2)...., T(v_n)$ are linear independent then $v_1, v_2, ...v_n$ are linear independent

(c) $T(v_1), T(v_2)...., T(v_n)$ are linear independent if T is onto

(d) None of these

45. The solution of $y' + y = xy^4$ is given by

(a) $y^{-3} = x + \frac{1}{3} + ce^{3x}$

(b) $y^3 = x + \frac{1}{3} + ce^{3x}$

(c) $y^{-3} = x + \frac{1}{3} + ce^{-3x}$

(d) $y^3 = x + \frac{1}{3} + ce^{-3x}$

46. The function $f(x) = |x|^p, 0 < p < 1$ is

(a) not Lipschitz at $x = 0$

(b) Lipschitz at $x = 0$ with Lipschitz constant 1

(c) Lipschitz at $x = 0$

(d) Differentiable at $x = 0$

47. The initial value problem $\frac{dx}{dt} = x^{3/2}(t), x(0) = 0$ has

(a) unique solution

(b) two solutions

(c) infinitely many solutions

(d) none of these

48. If u is a function of x, y and z satisfies the partial differential equation $(y-z)\frac{\partial u}{\partial x} + (z-x)\frac{\partial u}{\partial y} + (x-y)\frac{\partial u}{\partial x} = 0$.

Then the general soluiton is of the form

(a) $u = f(x+y+z, x^2+y^2+z^2)$

(b) $u = f(xy+yz+zx, x^2+y^2+z^2)$

(c) $u = f(xyz, x^2+y^2+z^2)$

(d) $u = f(x+y+z, x^2y^2z^2)$

49. The complete integral of the equation $(p+q)(z-xp-yq) = 1$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ is

(a) $z = ax + by + \frac{1}{a-b}$

(b) $z = ax + by + \frac{1}{a+b}$

(c) $z = ax - by + \frac{1}{a+b}$

(d) none of these

50. The orthogonal trajectories of the family of rectangular hyperbolas $y = c_1 / x$ is

(a) $y^2 - x^2 = c$

(b) $y^2 + x^2 = c$

(c) $x^2y^2 = c$

(d) $\frac{x^2}{y^2} = c$