

- (iv)  $\eta$  is one only when  $\frac{T_C}{T_H}$  is zero. i.e. when  $T_C = 0\text{K}$  or  $T_H = \infty\text{K}$ . Both these temperature are impossible to obtain. Therefore, the efficiency of an engine can never be one or 100%. That is heat can't be transformed completely into work.
- (v) For all reversible cycle operating between the same source and sink temperature, the efficiency is the same.

### Comparison of efficiencies of reversible and irreversible engine.

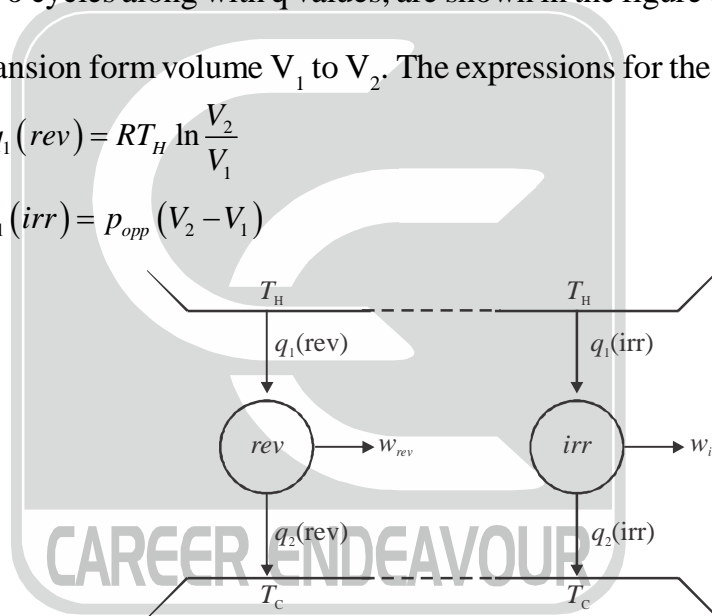
The efficiency of a reversible Carnot cycle is the theoretically possible maximum value which an engine can have. Since the various processes of this type of engine are to be carried out reversibly, therefore, such type of an engine does not have any realistic basis because reversible processes are idealized concepts which can never be realized. A real heat engine, which is irreversible in nature, will have efficiency smaller than the reversible heat engine.

Let us have two cycles, one operating reversibly and the other irreversibly. Let both of them operate between the same two temperature  $T_C$  and  $T_H$  and involve ideal gas as the working substance. These two cycles along with  $q$  values, are shown in the figure below.

(A) Isothermal expansion from volume  $V_1$  to  $V_2$ . The expressions for the work involved are

$$-w_1(\text{rev}) = q_1(\text{rev}) = RT_H \ln \frac{V_2}{V_1}$$

And  $-w_1(\text{irr}) = q_1(\text{irr}) = p_{\text{opp}}(V_2 - V_1)$



Since we know that  $|w_1(\text{rev})| > |w_1(\text{irr})|$ , therefore,

$$q_1(\text{rev}) > q_1(\text{irr})$$

(B) Isothermal compression from volume  $V_3$  to  $V_4$ . The expressions for the work involved are

$$-w_3(\text{rev}) = q_2(\text{rev}) = RT_C \ln \frac{V_4}{V_3}$$

$$-w_3(\text{irr}) = q_2(\text{irr}) = P'_{\text{ext}}(V_4 - V_3)$$

Now, since in the irreversible process, more work is done as compared to that in the reversible process, we have

$$w_3(\text{irr}) > w_3(\text{rev})$$

It follows that

$$|q_2(\text{irr})| > |q_2(\text{rev})|$$

Now the efficiencies of the two cycles are

$$\eta(\text{rev}) = \frac{q_1(\text{rev}) + q_2(\text{rev})}{q_1(\text{rev})} = 1 - \frac{|q_2(\text{rev})|}{q_1(\text{rev})}$$

$$\eta(\text{irr}) = \frac{q_1(\text{irr}) + q_2(\text{irr})}{q_1(\text{irr})} = 1 - \frac{|q_2(\text{irr})|}{q_1(\text{irr})}$$

Now since  $q_1(\text{rev}) > q_1(\text{irr})$  and  $|q_2(\text{rev})| < |q_2(\text{irr})|$ , therefore, it follows that

$$\frac{|q_2(\text{rev})|}{q_1(\text{rev})} < \frac{|q_2(\text{irr})|}{q_1(\text{irr})} \quad \text{or} \quad \left\{ 1 - \frac{|q_2(\text{rev})|}{q_1(\text{rev})} \right\} > \left\{ 1 - \frac{|q_2(\text{irr})|}{q_1(\text{irr})} \right\}$$

i.e.  $\eta(\text{rev}) > \eta(\text{irr})$

**Basic Conclusion from Efficiency of a Carnot Cycle :**

For a reversible Carnot cycle operating between two temperatures  $T_H$  and  $T_C$ , the efficiency is given as

$$\eta = \frac{q_1 + q_2}{q_1} = \frac{T_H - T_C}{T_H}$$

where  $q_1$  and  $q_2$  are the heats exchanged with the thermal reservoirs at temperatures  $T_H$  and  $T_C$ , respectively. Rewriting the above expression, we have

Or,  $1 + \frac{q_2}{q_1} = 1 - \frac{T_C}{T_H}$  or  $\frac{q_2}{q_1} = -\frac{T_C}{T_H}$

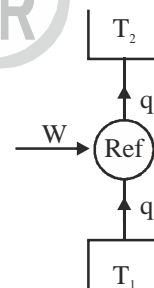
Or,  $\frac{q_1}{T_H} + \frac{q_2}{T_C} = 0$

that is, the sum of the ratios of the heat involved and the corresponding temperature is zero for a Carnot cycle.

**Carnot Refrigerator:**

It is the reverse of Carnot engine i.e. the energy flow from low temperature body to a high temperature body by providing energy in the form of work to the system. It is the energy transfer device therefore, the ratio of its output to input is represented by coefficient of performance which can be greater than 1.

In case of Carnot refrigerator system absorbed heat from low temperature body and transfer it to the high temperature body. In Carnot engine heat is input work is output. In refrigerator heat is output and work is input.



**Co-efficient of performance ( $\beta$ ) of Carnot refrigerator:**

It is defined as the ratio of heat transferred from a lower temperature to a higher temperature to the work done

on the system, i.e.  $\beta = \frac{|q_c|}{w}$

The lesser the work done the more efficient the operation and greater the coefficient of performance.

$$\beta = \frac{|q_c|}{|q_h| - |q_c|} = \frac{T_C}{T_H - T_C}$$