12. If the system of equation $x - ky - z = 0$, $kx - y - z = 0$ and $x + y - z = 0$ has a non zero solution, then the possible value of k are

(a) $-1, 2$ (b) $0, 1$ (c) $1, 2$ (d) $-1, 1$ **[IIT-JEE : 2011]**

Soln. Given that, system of equation is

$$x - ky - z = 0$$
$$kx - y - z = 0$$
$$x + y - z = 0$$

Matrix representation is $A = \begin{bmatrix} 1 & -k & -1 \\ k & -1 & -1 \\ 1 & 1 & -1 \end{bmatrix}$

Given system has non zero solution $\Rightarrow |A| = 0$

$\Rightarrow 1(1+1) + k(-k+1) - 1(k+1) = 0$

$\Rightarrow 2 - k^2 + \not{k} - \not{k} - 1 = 0 \Rightarrow -k^2 + 1 = 0$

$\Rightarrow k^2 = 1 \Rightarrow \boxed{k = \pm 1}$

Correct option is (d)

13. The equation has **[TIFR-2012]**

$$x_1 + \frac{1}{2}x_2 + \frac{1}{3}x_3 = 1$$
$$x_1 + \frac{1}{4}x_2 + \frac{1}{9}x_3 = 1$$
$$x_1 + \frac{1}{8}x_2 + \frac{1}{27}x_3 = 1$$

has no solution. (T/F).

Soln. After solving these we get $\boxed{\rho(A) = \rho(A:B) = 3}$, hence unique solution

Statement is false.

14. The equation $x_1 + 2x_2 + 3x_3 = 1, \quad x_1 + 4x_2 + 9x_3 = 1, \quad x_1 + 8x_2 + 27x_3 = 1$ have **[TIFR-2010]**

(a) only one solution (b) two solution
(c) infinitely many solution (d) no solution

Soln. $\boxed{\rho(A) = \rho(A:B)}$ and also A is invertible

Correct option is (a)

15. Let A be a 5×4 matrix with real entries such that $A\underline{x} = \underline{0}$ iff $\underline{x} = \underline{0}$ where $\underline{x}$ is 4×1 vector and $\underline{0}$ is null vector. Then rank of A is **[(SCQ) CSIR-NET/JRF : Dec. 2013]**

(a) 4 (b) 5 (c) 2 (d) 1

Soln. $Ax = 0$ has trivial solution only If $\rho(A)$ = the number of columns = 4

Correct option is (a).

16. Let A be a 5×4 matrix with real entries such that the space of all solution of the linear system. $Ax^t = [1, 2, 3, 4, 5]^t$ is given by $\left\{[1+2s, 2+3s, 3+4s, 4+5s]^t : s \in \mathbb{R}\right\}$. Here M^t denote the transpose of a matrix M) then the rank of A is equal to **[(SCQ) CSIR-NET/JRF : Dec-2011]**

(a) 4 (b) 3 (c) 2 (d) 1

Soln. Solution set of $Ax^t = [1\ 2\ 3\ 4\ 5]^t$ is given by

$$S = \left\{[1+2s, 2+3s, 3+4s, 4+5s]^t : s \in \mathbb{R}\right\} = \left\{(1,2,3,4) + s(2,3,4,5) : s \in \mathbb{R}\right\} \Rightarrow \dim S = 2$$

$\therefore$ The number of L.I. solution of the given non homogeneous system of equation is 2.

Hence, $n - r + 1 = 2$

$\Rightarrow 4 - r + 1 = 2 \quad \Rightarrow \not{-}r = \not{-}3 \quad \Rightarrow \boxed{r = 3}$

Correct option is (b).

17. $[A]_{m\times n}\ x=[b]_{n\times 1}$ be a system of equation then which of the following is true?

(a) If $m=n$ then the given system of equation has always unique solution **[(SCQ) CSIR-NET/JRF : Dec-2011]**

(b) If $m<n$ then the given system of equation has no solution

(c) If $m<n$ then the given system of equation has infinite or no solution

(d) If $m\le n$ then the given system of equation has no solution

Soln. We know that, if number of equation is less than number of unknowns then system of linear equation have either no or infinite solutions. As, rank $[A:B]<$ number of unknowns and if rank $[A:B]$ = rank $[A]\Rightarrow$ infinite solution. rank $[A:B]\ne$ rank $[A]$ $\Rightarrow$ no solution.

Correct option is (c).

18. Tick true or false

A is 3×4 matix of rank 3. Then system of equations $Ax=b$ has exactly one solution. **[TIFR-2011]**

Soln. $n-r=4-3=1$ so it has infinite number of solution.

Statement is false.

Unsolved Problems

SIMULTANEOUS LINEAR SYSTEM

In all the following, you show consistency and inconsistency. If consistent find solution on it.

(1) $x+2y-3z=-4$, $2x+3y+2z=2$, $3x-3y-4z=1$

(2) $x+y+z=6$, $x+2z=7$, $3x+y+z=12$

(3) $5x-7y=2$, $7x-5y=3$

(4) $4x-2y=3$, $6x-3y=5$

(5) $x-2y+z=0$, $x+y-z=0$, $3x+6y-5z=0$

(6) $4x+3z=8$, $2x-z=2$, $3x+2y=5$

(7) $x+y+z=5$, $x+3y+3z=9$, $x+2y+\alpha z=\beta$ Find α,β so that it have infinite many solution.

(8) $x+y+z=3$, $x+2y+3z=4$, $x+4y+kz=6$ will not have unique solution for k equal to

(a) 0 (b) 5 (c) 6 (d) 7

(9) $x+2y+z=6$, $2x+y+2z=6$, $x+y+z=5$

(10) $2x+3y=4$, $x+y+z=4$, $x+2y-z=a$ for what value of a thesystem has solution

(11) $4x+2y=7$, $2x+y=6$

(12) $-x+5y=-1$, $x-y=2$, $x+3y=3$

(13) $2x_1-x_2+3x_3=1$, $3x_1-2x_2+5x_3=2$, $-x_1-4x_2+x_3=3$

(14) $x_1+x_2+2x_3=1$, $x_1+2x_2+3x_3=2$, $x_1+4x_2+ax_3=4$ has unique solution. Find a = ?

3 Vector Spaces

Binary Operation: Let G be a non-empty set. Then $G \times G = \{(a,b) : a, b \in G\}$.

If $f : G \times G \to G$ is a function, then f is called the binary operation on G.

Let '$*$' be the binary operation on G if and only if $a * b \in G \; \forall \, a, b \in G$ and $a * b$ is unique.

This is called closure property of binary operation and set G is closed with respect to binary operation.

Group: Let $(G, \oplus)$ be an algebraic structure where $\oplus$ is binary operation over non-empty set G, then $(G, \oplus)$ is called group under the operation $\oplus$ if following postulate are satisfied:

1. **Closure Property:** The binary operation $\oplus$ is closed. i.e. $a \oplus b \in G$ for all $a, b \in G$
2. **Associative Law:** The binary operation $\oplus$ follow associative law.

   i.e. $a \oplus (b \oplus c) = (a \oplus b) \oplus c$ for all $a, b, c \in G$
3. **Existence of Identity Element:** There exist an element $e \in G$ called identity such that $a \oplus e = e \oplus a = a \, \forall a \in G$
4. **Existence of Inverse:** For each element 'a' in G, there exist an element a^{-1} (inverse of a) is G, such that $a \oplus a^{-1} = a^{-1} \oplus a = e$

Remark 1: Many author do not mention first property as this is same as of the definition of binary operation.

Abelian Group or Commutative Group: A group G is said to be abelian if commutative law holds i.e. $a \oplus b = b \oplus a \, \forall a, b \in G$.

Remark 2: A group with addition binary operation is known as additive group.

Remark 3: A group with multiplication binary operation is known as multiplicative group.

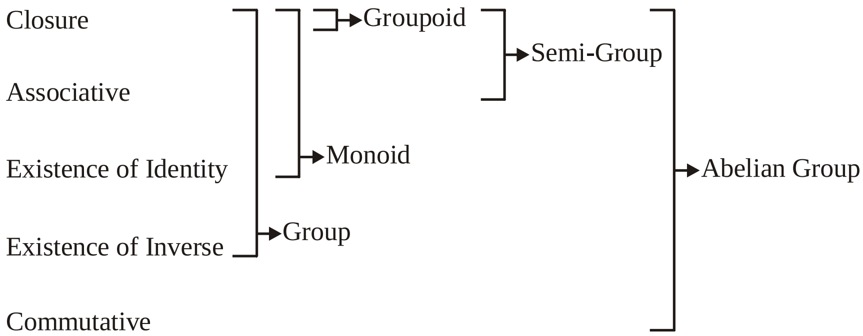
Groupoid: Let $(S, \oplus)$ be an algebraic structure and $\oplus$ is binary operation over non-empty set S. $(S, \oplus)$ is called groupoid if it follow closure property.

Semi-group: Let S be a non-empty set, then $(S, \oplus)$ is called semi-group if it satisfies following property:

1. **Closure Property:** $a \oplus b \in S \, \forall a, b \in S$
2. **Associative Property:** $a \oplus (b \oplus c) = (a \oplus b) \oplus c \, \forall a, b, c \in S$

Monoid: Let S be a non-empty set, $(S, \oplus)$ is called monoid if it satisfies following property:

1. **Closure Property:** $a \oplus b \in S \, \forall a, b \in S$
2. **Associative Property:** $a \oplus (b \oplus c) = (a \oplus b) \oplus c \, \forall a, b, c \in S$
3. **Existence of Identity:** There exist $e \in G$ such that $a \oplus e = e \oplus a = a \, \forall a \in G$



Field: An algebraic structure (F, +, .) having at least two elements is said to be field if it possesses following properties.

1. (F, +) is abelian group.

2. (F', .) is abelian group, where $F' = F - \{0\}$

3. Multiplication distribute over addition i.e.

$a.(b+c) = ab + ac \; \forall a, b, c \in F$

OR (F, +, .) is called field if

F_1 – Addition is closed i.e., $a + b \in F, \forall a, b \in F$

F_2 – Addition is associative i.e., $a + (b + c) = (a + b) + c, \forall a, b, c \in F$

F_3 – $\exists$ element $0 \in F$ such that $a + 0 = a, \forall a \in F$

F_4 – For each element $a \in F \; \exists (-a) \in F$ such that $a + (-a) = 0, \forall a \in F$

F_5 – Addition is commutative i.e., $a + b = b + a, \; \forall a, b \in F$

F_6 – Multiplication is closed i.e., $a.b \in F \; \forall a, b \in F$

F_7 – Multiplication is associative i.e., $a.(b.c) = (a.b).c, \forall a, b, c \in F$

F_8 – $\exists$ a non-zero element denoted by $1 \in F$ such that $a.1 = 1.a = a, \forall a \in F$

F_9 – For every non-zero element $a \in F$, $\exists a^{-1} \in F$ such that $a.a^{-1} = 1, \; \forall a \in F$

F_{10} – Multiplication is commutative i.e. $a.b = b.a \; \forall a.b \in F$

F_{11} – Multiplication is distributive over addition i.e.

$a.(b + c) = a.b + a.c$ and $\forall a, b, c \in F$

'0' identity element of addition is called zero-element of field, '1' identity element of multiplication is called unity of field.

Vector space: A vector space (or linear space) consists of the following:

1. *A* field F of scalars

2. *A* set V of objects, called vectors

3. *A* rule (or operation) called vector addition and a rule (or operation), called scalar multiplication.

Definition: A non-empty set V is said to be a vector space over the field F if the following conditions hold:

Vector addition:

(a) Addition is closed i.e. for all $\alpha, \beta \in V, \alpha + \beta \in V$

(b) Addition is commutative, $\alpha + \beta = \beta + \alpha$

(c) Addition is associative $\alpha + (\beta + \gamma) = (\alpha + \beta) + \gamma$

(d) There is a unique vector 0 in V, called the zero vector, such that $\alpha + 0 = \alpha$ for all $\alpha \in V$