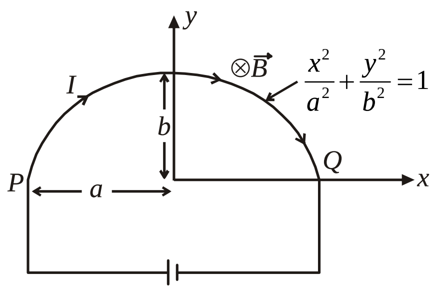
The period of rotation is $T = \dfrac{2\pi m}{qB} = \dfrac{2\times 3.14\times 1.67\times 10^{-27}}{1.6\times 10^{-19}\times 0.3} = 21.85\times 10^{-8}$ sec.

Therefore, pitch of the helix is $p = v_{||}\times T = (2\times 10^5 \text{ m/s})\,(21.85\times 10^{-8}\text{ s}) = 43.7\times 10^{-3}\text{ m} = 43.7\text{ mm}$

Example: In figure shown there exists a uniform magnetic field $\vec{B}$ in the region. Calculate net force on the wire PQ due to the magnetic field.



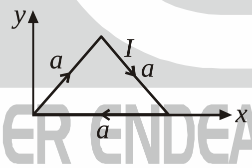
Soln. Force on the wire is $\vec{F} = \int I\, d\vec{l}\times\vec{B}$

$$= I\int\left(dx\hat{i} + dy\hat{j}\right)\times B\left(-\hat{k}\right) = I\,B\int\left(dx\hat{j} - dy\hat{i}\right)$$

$$= IB\left[\hat{j}\int_{-a}^{+a} dx - \hat{i}\int_{0}^{0} dy\right] = 2IBa\hat{j}$$

Example: In the figure shown there exists a non uniform magnetic field $\vec{B} = x\hat{k}$. claculate force on the circular loop shown in figure.

Soln. Magnetic force on the loop is given as



$$\vec{F} = \oint I\, d\vec{l}\times\vec{B}$$

$$= I\oint\left(dx\hat{i} + dy\hat{j}\right)\times k\,x\hat{k}$$

$$= Ik\left[\int xdx\left(-\hat{j}\right) + \int xdy\left(\hat{i}\right)\right] = Ik\left[-\hat{j}\int_0^0 xdx + \hat{i}\int_0^0 xdy\right]$$

$$= Ik\left\{-\hat{j}\left[\frac{x^2}{2}\right]_0^0 + \hat{i}\left[\int_0^{a/2}\frac{y}{\sqrt{3}}dy + \int_{a/2}^{0}\left(a - \frac{y}{\sqrt{3}}\right)dy + \int_a^0 0\, dy\right]\right\}$$

$$= Ik\left\{0 + \hat{i}\left[\frac{a^2}{8\sqrt{3}} - \frac{a^2}{2} + \frac{a^2}{8\sqrt{3}}\right]\right\} = \frac{-Ika^2}{4}\left[2 - \frac{1}{\sqrt{3}}\right]\hat{i}$$

Example: Find the value of the magnetic field required to maintain non-relativistic protons of energy 1 MeV in a circular orbit of radius 100 mm.

(Given: $m_p = 1.67\times 10^{-27} kg,\ e = 1.6\times 10^{-19} C$

Soln. $\frac{1}{2}mv^2 = 1\times 10^6\times 1.6\times 10^{-19}\, J$

$$\Rightarrow \qquad v=\sqrt{\frac{3.2\times10^{-13}}{m}}=\sqrt{\frac{3.2\times10^{-13}}{1.67\times10^{-27}}}=\sqrt{1.91677\times10^{14}}=1.384474\times10^{7}\ m/\sec$$

Now, for circular orbit

$$q\,vB=\frac{mv^2}{r}$$

$$B=\frac{mv}{qr}=\frac{1.67\times10^{-27}\times1.384474\times10^{7}}{1.6\times10^{-19}\times10^{-1}}$$

$$=\frac{1.67\times1.384474}{1.6}=1.4450\text{ Tesla}$$

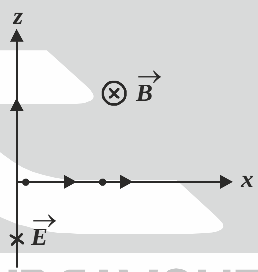
Example: A particle of mass 1×10^{-26} kg and charge $+1.6\times10^{-19}$ C travelling with a velocity 1.28×10^{6} m/s in the +x-direction enters a region in which a uniform electric field E and a uniform magnetic field of induction B are present such that $E_x=E_y=0,\ E_z=102.4$ kV/m and $B_x=B_z=0,\ B_y=8\times10^{-2}$ Wb m^{-2}. The particle enters this region at the origin at time $t=0$. Determine the location x, y and z coordinates of the particle at $t=5\times10^{-6}$ s. If the electric field is switched off at this instant (with magnetic field still present), what will be the position of the particle at $t=7.45\times10^{-6}$ s ?

Soln. Velocity of particle in +x-direction $=1.28\times10^{6}$ m/sec.

$E_x=E_y=0,\ E_z=102.4$ kV/m

$B_x=B_z=0,\ B_y=8\times10^{-2}$ Wb m^{-2}

These are shown in figure



Electric force $\vec{F}_e$ on the charge is given by

$$\left|\vec{F}_e\right|=q\,\vec{E}=(1.6\times10^{-19})\,(102.4\times10^{3})$$

$=163.84\times10^{-16}$ along –z-direction

Magnetic force $\vec{F}_m$ on the charge is given

$$\left|\vec{F}_m\right|=q\,v\,B$$

$$=(1.6\times10^{-19})\,(1.28\times10^{6})\,(8\times10^{-2})$$

$=163.84\times10^{-16}$ along +z-direction

Thus there are two equal, opposite and collinear forces acting on the particle along z-axis. Hence the resultant force on particle is zero. The particle moves thus along the x-axis without deflection.

(a) At time $t=5\times10^{-6}$ s, the distance x travelled by the particle.

$$x=v\,t=(1.28\times10^{6})\,(5\times10^{-6})=6.4\text{ m}$$

Therefore, coordinates of the particle = (6.4, 0, 0).

(b) When the electric field is switched off, there will be a force 163.84×10^{-16} along +z-axis acting on the particle moves in uniform magnetic field and describes a circular path in xz-plane as shown in figure. We know that when a particle of mass m and charge q is subjected to a magnetic field B acting perpendicular to it, the particle moves with velocity v in a circular trajectory whose radius r is given by

$$B\,q\,v=m\,v^2/r\ \text{ or }\ r=\frac{m\,v}{B\,q}$$