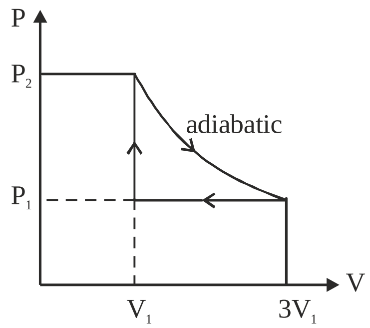
Problem : Consider a monoatomic ideal gas operating in a closed cycle as shown in the P-V diagram given below. The ratio $\frac{P_1}{P_2}$ is ______________ (upto two place decimal)



(Specify your answer upto two digits after the decimal point)

Soln. For adiabatic process,

$$P_2V_1^{\gamma} = P_1(3V_1)^{\gamma}$$

where, $\gamma = \frac{5}{3}$ (for monoatomic gas)

$$\therefore \quad \frac{P_2}{P_1} = 3^{\gamma} = 3^{5/3}$$

or, $$\frac{P_1}{P_2} = 3^{-5/3} = 0.1602 = 0.16 \text{ (upto two place decimal)}$$

Correct answer is (0.16)

Problem : A motor car tyre has a pressure of 2 atm at room temperature 27°C. If the tyre suddenly bursts, find the resulting temperature.

Soln. Given : $P_1 = 2$ atm, $P_2 = 1$ atm, $T_1 = 27°C = 27 + 273 = 300$ K, $T_2 = ?$

Since the bursting of tyre happens suddenly, the process must be adiabatic and hence, we have

$$\frac{P_1^{\gamma-1}}{T_1^{\gamma}} = \frac{P_2^{\gamma-1}}{T_2^{\gamma}}$$

$$\Rightarrow \left(\frac{P_2}{P_1}\right)^{\gamma-1} = \left(\frac{T_2}{T_1}\right)^{\gamma}$$

$$\Rightarrow \left(\frac{1}{2}\right)^{1.4-1} = \left(\frac{T_2}{300}\right)^{1.4} \qquad \Rightarrow \left(\frac{1}{2}\right)^{0.4} = \left(\frac{T_2}{300}\right)^{1.4}$$

where we have taken the final pressure to be 1 atm.

Solving, we have

$$\Rightarrow \quad T_2 = 246.1 \text{ K} = -26.9°C$$

Problem : A quantity of air at 27°C and atmospheric pressure is suddenly compressed to half of its original volume. Find the (i) pressure and (ii) temperature.

Soln. Given $P_1 = 1$ atm, $P_2 = ?$, $\gamma = 1.4$

$V_1 = V$, $V_2 = V/2$, $T_1 = 27°C = 300K$, $T_2 = ?$

For an adiabatic process, $P_1V_1^\gamma = P_2V_2^\gamma$

$$\Rightarrow \quad P_2 = P_1\left(\frac{V_1}{V_2}\right)^\gamma = 1\left(\frac{V}{\frac{V}{2}}\right) = 1(2)^{1.4} = 2.636\ atm$$

Also, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$\Rightarrow \quad T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300\left(\frac{V}{\frac{V}{2}}\right)^{1.4-1} = 300(2)^{0.4} = 395.9\text{K} = 122.9^\circ\text{C}$$

Problem : 1 gm molecule of a manoatomic $\left(\gamma = \frac{5}{3}\right)$ perfect gas at 27°C adiabatically compressed in reversible process from an initial pressure 1 atmosphere to a final pressure of 50 atmosphere. Calculate the resulting difference in temperature.

Soln. Given: $P = 1$ atm, $P_2 = 50$ atm, $T_1 = 27^\circ$C = 300K, $T_2 = ?$, $\gamma = \frac{5}{3}$

Using $\frac{P_1^{\gamma-1}}{T_1^\gamma} = \frac{P_2^{\gamma-1}}{T_2^\gamma}$ or $\left(\frac{P_2}{P_1}\right)^{\gamma-1} = \left(\frac{T_2}{T_1}\right)^\gamma$

$$\Rightarrow \quad (50)^{\frac{5}{3}-1} = \left(\frac{T_2}{300}\right)^{\frac{5}{3}} \Rightarrow 50^{\frac{2}{3}} = \left(\frac{T_2}{300}\right)^{\frac{5}{3}}$$

$$\Rightarrow \quad \frac{2}{3}\log 50 = \frac{5}{3}\log T_2 - \frac{5}{3}\log 300$$

$$\Rightarrow \quad T_2 = 1434\ K = 1161\ ^oC$$

The required difference in temperature = 1161 °C – 27 °C = 1134 °C.

Problem : Three moles of an ideal monatomic gas occupy a volume of 20 m³ at 300 K. If the gas expands adiabatically to 40 m³ the final pressure is nearest to

(a) 331 N/m² (b) 1200 N/m² (c) 980 N/m²
(d) 486 N/m² (e) 118 N/m²

Soln. For an ideal gas, we have

$$P_i = \frac{nRT}{V_i} = \frac{3R \times 300}{20} = 374.13\ \text{N/m}^2.$$

Now, for an adiabatic process, we have

$$P_f = \left(\frac{V_i}{V_f}\right)^\gamma P_i, \text{ where } \gamma = \frac{5}{3} \text{ for monoatomic gas.}$$

$$\therefore \quad P_f = \frac{374.13}{2^{5/3}} = 117.84\ \text{N/m}^2$$

Correct option is (e)