PAPER : IIT-JAM

MATHEMATICS MA–2018

SECTION-A

[Multiple Choice Questions (MCQ)]

Q.1 – Q.10 carry ONE mark each.

1. The tangent plane to the surface $z=\sqrt{x^2+3y^2}$ at (1, 1, 2) is given by

(a) $x-3y+z=0$ (b) $x+3y-2z=0$

(c) $2x+4y-3z=0$ (d) $3x-7y+2z=0$

2. Let $f:\mathbb{R}^3\to\mathbb{R}$ be a scalar field, $\vec{v}:\mathbb{R}^3\to\mathbb{R}^3$ and let $\vec{a}\in\mathbb{R}^3$ be a constant vector. If $\vec{r}$ represents the position vector $x\hat{i}+y\hat{j}+z\hat{k}$, then which one of the following is FALSE?

(a) $curl(f\,\vec{v})=grad(f)\times\vec{v}+f\,curl(\vec{v})$ (b) $div(grad(f))=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)f$

(c) $curl(\vec{a}\times\vec{r})=2|\vec{a}|\vec{r}$ (d) $div\left(\frac{\vec{r}}{|\vec{r}|^3}\right)=0$, for $\vec{r}\neq\vec{0}$

3. In $\mathbb{R}^3$, the cosine of the acute angle between the surfaces $x^2+y^2+z^2-9=0$ and $z-x^2-y^2+3=0$ at the point (2, 1, 2) is

(a) $\frac{8}{5\sqrt{21}}$ (b) $\frac{10}{5\sqrt{21}}$ (c) $\frac{8}{3\sqrt{21}}$ (d) $\frac{10}{3\sqrt{21}}$

4. Let $s_n=1+\frac{1}{1!}+\frac{1}{2!}+...+\frac{1}{n!}$ for $n\in\mathbb{N}$. Then which one of the following is TRUE for the sequence $\{s_n\}_{n=1}^{\infty}$

(a) $\{s_n\}_{n=1}^{\infty}$ converges in $\mathbb{Q}$

(b) $\{s_n\}_{n=1}^{\infty}$ is a Cauchy sequence but does not converge in $\mathbb{Q}$

(c) The sub-sequence $\{s_{k^n}\}_{n=1}^{\infty}$ is convergent in $\mathbb{R}$ only when *k* is even natural number

(d) $\{s_n\}_{n=1}^{\infty}$ is not a Cauchy sequence

5. Consider the vector space *V* over $\mathbb{R}$ of polynomial functions of degree less than or equal to 3 defined on $\mathbb{R}$. Let $T:V\to V$ be defined by $(Tf)(x)=f(x)-xf'(x)$. Then the rank of T is

(a) 1 (b) 2 (c) 3 (d) 4

6. Let *a* be a positive real number. If *f* is a continuous and even function defined on the interval $[-a,a]$, then $\int_{-a}^{a}\frac{f(x)}{1+e^x}dx$ is equal to

(a) $\int_0^a f(x)dx$ (b) $2\int_0^a\frac{f(x)}{1+e^x}dx$ (c) $2\int_0^a f(x)\,dx$ (d) $2a\int_0^a\frac{f(x)}{1+e^x}dx$

7. In $\mathbb{R}^2$, the family of trajectories orthogonal to the family of asteroids $x^{2/3}+y^{2/3}=a^{2/3}$ is given by

(a) $x^{4/3}+y^{4/3}=c^{4/3}$ (b) $x^{4/3}-y^{4/3}=c^{4/3}$ (c) $x^{5/3}-y^{5/3}=c^{5/3}$ (d) $x^{2/3}-y^{2/3}=c^{2/3}$

8. If (v_1,v_2,v_3) is a linearly independent set of vectors in a vector space over $\mathbb{R}$, then which one of the following sets is also linearly independent?

(a) $\{v_1+v_2-v_3, 2v_1+v_2+3v_3, 5v_1+4v_2\}$

(b) $\{v_1-v_2, v_2-v_3, v_3-v_1\}$

(c) $\{v_1+v_2-v_3, v_2+v_3-v_1, v_3+v_1-v_2, v_1+v_2+v_3\}$

(d) $\{v_1+v_2, v_2+2v_3, v_3+3v_1\}$

9. Which one of the following is TRUE?

(a) $\mathbb{Z}_n$ is cyclic if and only if n is prime (b) Every proper sub-group of $\mathbb{Z}_n$ is cyclic

(c) Every proper sub-group of S_4 is cyclic

(d) If every proper sub-group of a group is cyclic, then the group is cyclic

10. Let $a_n=\frac{b_{n+1}}{b_n}$, where $b_1=1$, $b_2=1$ and $b_{n+2}=b_n+b_{n+1}, n\in\mathbb{N}$. Then $\lim\limits_{n\to\infty}a_n$ is

(a) $\frac{1-\sqrt{5}}{2}$ (b) $\frac{1-\sqrt{3}}{2}$ (c) $\frac{1+\sqrt{3}}{2}$ (d) $\frac{1+\sqrt{5}}{2}$

Q.11 – Q.30 carry TWO marks each.

11. For $x\in\mathbb{R}$, let $f(x)=\begin{cases} x^3\sin(\frac{1}{x}), & x\neq 0 \\ 0, & x=0\end{cases}$. Then which one of the following is FALSE?

(a) $\lim\limits_{x\to 0}\frac{f(x)}{x}=0$ (b) $\lim\limits_{x\to 0}\frac{f(x)}{x^2}=0$

(c) $\frac{f(x)}{x^2}$ has infinitely many maxima and minima on the interval (0, 1)

(d) $\frac{f(x)}{x^4}$ is continuous at $x=0$ but not differentiable at $x=0$

12. If $\vec{F}(x,y)=(3x-8y)\hat{i}+(4y-6xy)\hat{j}$ for $(x,y)\in\mathbb{R}^2$, then $\oint_C \vec{F}\cdot d\vec{r}$, where C is the boundary of the triangular region bounded by lines $x=0$, $y=0$ and $x+y=1$ oriented in the anti-clockwise directions, is

(a) $\frac{5}{2}$ (b) 3 (c) 4 (d) 5

13. Let $f(x,y)=\begin{cases} \frac{xy}{(x^2+y^2)^{\alpha}}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0)\end{cases}$. Then which one of the following is TRUE for f at the point (0, 0)?

(a) For $\alpha=1$, f is continuous but not differentiable

(b) For $\alpha=\frac{1}{2}$, f is continuous and differentiable

(c) For $\alpha=\frac{1}{4}$, f is continuous and differentiable

(d) For $\alpha=\frac{3}{4}$, f is neither continuous nor differentiable

14. Let $a,b,c \in \mathbb{R}$. Which of the following values of a,b,c do NOT result in the convergence of the series $\sum_{n=3}^{\infty} \frac{a^n}{n^b (\log_e n)^c}$?

(a) $|a|<1, b \in \mathbb{R}, c \in \mathbb{R}$
(b) $a=1, b>1, c \in \mathbb{R}$
(c) $a=1, b \ge 0, c<1$
(d) $a=-1, b \ge 0, c>0$

15. Let $f:\mathbb{R} \to \mathbb{R}$ be a function and let J be a bounded open interval in $\mathbb{R}$.

Define $W(f,J)=\sup\{f(x) \mid x \in J\} - \inf\{f(x) \mid x \in J\}$. Which one of the following is FALSE?

(a) $W(f,J_1) \le W(f,J_2)$ if $J_1 \subset J_2$

(b) If f is a bounded function in J and $J \supset J_1 \supset J_2 ... \supset J_n \supset ...$ such that the length of the interval J_n tends to 0 as $n \to \infty$, then $\lim_{n\to\infty} W(f,J_n)=0$

(c) If f is discontinuous at a point $a \in J$, then $W(f,J) \ne 0$

(d) If f is continuous at a point $a \in J$, then for any given $\in > 0$ there exists an interval $I \subset J$ such that $W(f,I) < \in$

16. Let U, V and W be finite dimensional real vector spaces, $T:U \to V$, $S:V \to W$ and $P:W \to U$ be linear transformations. If range (ST) = nullspace (P), nullspace (ST) = range (P) and rank (T) = rank (S), then which one of the following is TRUE?

(a) nullity of T = nullity of S
(b) dimension of U $\ne$ dimension of W
(c) If dimension of V = 3, dimension of U = 4, then P is not identically zero
(d) If dimension of V = 4, dimension of U = 3, and T is one-one, then P is identically zero

17. Let $a_n = \frac{(-1)^n}{\sqrt{1+n}}$ and let $c_n = \sum_{k=0}^{n} a_{n-k} a_k$, where $n \in \mathbb{N} \cup \{0\}$. Then which one of the following is TRUE?

(a) Both $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=1}^{\infty} c_n$ are convergent

(b) $\sum_{n=0}^{\infty} a_n$ is convergent but $\sum_{n=1}^{\infty} c_n$ is not convergent

(c) $\sum_{n=1}^{\infty} c_n$ is convergent but $\sum_{n=0}^{\infty} a_n$ is not convergent

(d) Neither $\sum_{n=0}^{\infty} a_n$ nor $\sum_{n=1}^{\infty} c_n$ is convergent

18. Let G be a group satisfying the property that $f: G \to \mathbb{Z}_{221}$ is a homomorphism implies $f(g)=0, \forall g \in G$. Then a possible group G is

(a) $\mathbb{Z}_{21}$
(b) $\mathbb{Z}_{51}$
(c) $\mathbb{Z}_{91}$
(d) $\mathbb{Z}_{119}$

19. An integrating factor of the differential equation $\left(y+\frac{1}{3}y^3+\frac{1}{2}x^2\right)dx+\frac{1}{4}(x+xy^2)dy=0$ is

(a) x^2
(b) $3\log_e x$
(c) x^3
(d) $2\log_e x$

20. Let H be the quotient group $\mathbb{Q}/\mathbb{Z}$. Consider the following statements.

I. Every cyclic subgroup of H is finite

II. Every finite cyclic group is isomorphic to a subgroup of H

Which one of the following holds?

(a) I is TRUE but II is FALSE (b) II is TRUE but I is FALSE

(c) both I and II are TRUE (d) neither I nor II is TRUE

21. Let $a_n = \begin{cases} 2 + \dfrac{(-1)^{\frac{n-1}{2}}}{n}, & \text{if } n \text{ is odd} \\ 1 + \dfrac{1}{2^n}, & \text{if } n \text{ is even} \end{cases}$, $n \in \mathbb{N}$. Then which one of the following is TRUE?

(a) $\sup\{a_n \mid n \in \mathbb{N}\} = 3$ and $\inf\{a_n \mid n \in \mathbb{N}\} = 1$ (b) $\lim\inf(a_n) = \lim\sup(a_n) = \dfrac{3}{2}$

(c) $\sup\{a_n \mid n \in \mathbb{N}\} = 2$ and $\inf\{a_n \mid n \in \mathbb{N}\} = 1$ (d) $\lim\inf(a_n) = 1$ and $\lim\sup(a_n) = 3$

22. Let $a_n = n + \dfrac{1}{n}, n \in \mathbb{N}$. Then the sum of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \dfrac{a_{n+1}}{n!}$ is

(a) $e^{-1} - 1$ (b) e^{-1} (c) $1 - e^{-1}$ (d) $1 + e^{-1}$

23. For $x > \dfrac{-1}{2}$, let $f_1(x) = \dfrac{2x}{1+2x}$, $f_2(x) = \log_e(1+2x)$ and $f_3(x) = 2x$. Then which one of the following is TRUE?

(a) $f_3(x) < f_2(x) < f_1(x)$ for $0 < x < \dfrac{\sqrt{3}}{2}$ (b) $f_1(x) < f_3(x) < f_2(x)$ for $x > 0$

(c) $f_1(x) + f_2(x) < \dfrac{f_3(x)}{2}$ for $x > \dfrac{\sqrt{3}}{2}$ (d) $f_2(x) < f_1(x) < f_3(x)$ for $x > 0$

24. Consider the group $\mathbb{Z}^2 = \{(a,b) \mid a, b \in \mathbb{Z}\}$ under component wise addition. Then which of the following is a subgroup of $\mathbb{Z}^2$?

(a) $\{(a,b) \in \mathbb{Z}^2 \mid ab = 0\}$ (b) $\{(a,b) \in \mathbb{Z}^2 \mid 3a + 2b = 15\}$

(c) $\{(a,b) \in \mathbb{Z}^2 \mid 7 \text{ divides } ab\}$ (d) $\{(a,b) \in \mathbb{Z}^2 \mid 2 \text{ divides } a \text{ and } 3 \text{ divides } b\}$

25. A particular integral of the differential equation $y'' + 3y' + 2y = e^{e^x}$ is

(a) $e^{e^x} e^{-x}$ (b) $e^{e^x} e^{-2x}$ (c) $e^{e^x} e^{2x}$ (d) $e^{e^x} e^{x}$

26. $a, b \in \mathbb{R}$ and let $f: \mathbb{R} \to \mathbb{R}$ be a thrice differential function. If $z = e^u f(v)$, where $u = ax + by$ and $v = ax - by$, then which one of the following is TRUE?

(a) $b^2 z_{xx} - a^2 z_{yy} = 4a^2 b^2 e^u f'(v)$ (b) $b^2 z_{xx} - a^2 z_{yy} = -4e^u f'(v)$

(c) $bz_x + az_y = abz$ (d) $bz_x + az_y = -abz$

27. Let $y(x)$ be the solution of the differential equation $\frac{dy}{dx}+y=f(x)$, for $x\geq 0$, $y(0)=0$, where $f(x)=\begin{cases}2, & 0\leq x<1\\ 0, & x\geq 1\end{cases}$. Then $y(x)=$

(a) $2(1-e^{-x})$ when $0\leq x<1$ and $2(e-1)e^{-x}$ when $x\geq 1$

(b) $2(1-e^{-x})$ when $0\leq x<1$ and 0 when $x\geq 1$

(c) $2(1-e^{-x})$ when $0\leq x<1$ and $2(1-e^{-1})e^{-x}$ when $x\geq 1$

(d) $2(1-e^{-x})$ when $0\leq x<1$ and $2e^{1-x}$ when $x\geq 1$

28. Consider the region D in the yz plane bounded by the line $y=\frac{1}{2}$ and the curve $y^2+z^2=1$, where $y\geq 0$. If the region D is revolved about the z-axis in $\mathbb{R}^3$, then the volume of the resulting solid is

(a) $\frac{\pi}{\sqrt{3}}$ (b) $\frac{2\pi}{\sqrt{3}}$ (c) $\frac{\pi\sqrt{3}}{2}$ (d) $\pi\sqrt{3}$

29. Suppose that $f,g:\mathbb{R}\to\mathbb{R}$ are differentiable functions such that f is strictly increasing and g is strictly decreasing. Define $p(x)=f(g(x))$ and $q(x)=g(f(x))$, $\forall x\in\mathbb{R}$. Then for $t>0$ the sign of $\int_0^t p'(x)(q'(x)-3)dx$ is

(a) positive (b) negative (c) dependent on t (d) dependent on f and g

30. Let I denote the 4 × 4 identity matrix. If the roots of the characteristic polynomial of a 4 × 4 matrix M are $\pm\sqrt{\frac{1\pm\sqrt{5}}{2}}$, then $M^8=$

(a) $I+M^2$ (b) $2I+M^2$ (c) $2I+3M^2$ (d) $3I+2M^2$

SECTION-B

[Multiple Select Questions (MSQ)]

Q.31 – Q.40 carry TWO marks each.

31. The solution(s) of the differential equation $\frac{dy}{dx}=(\sin 2x)y^{1/3}$ satisfying $y(0)=0$ is(are)

(a) $y(x)=0$

(b) $y(x)=-\sqrt{\frac{8}{27}}\sin^3 x$

(c) $y(x)=\sqrt{\frac{8}{27}}\sin^3 x$

(d) $y(x)=\sqrt{\frac{8}{27}}\cos^3 x$

32. Let P and Q be two non-empty disjoint subsets of $\mathbb{R}$. Which of the following is (are) FALSE?

(a) If P and Q are compact, then $P\cup Q$ is also compact

(b) If P and Q are not connected, then $P\cup Q$ is also not connected

(c) If $P\cup Q$ and P are closed, then Q is closed

(d) If $P\cup Q$ and P are open, then Q is open

33. Let $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$ denote the group of non-zero complex numbers under multiplication. Suppose $Y_n = \{z \in \mathbb{C} \mid z^n = 1\}, n \in \mathbb{N}$. Which of the following is (are) subgroup(s) of $\mathbb{C}^*$?

(a) $\bigcup_{n=1}^{100} Y_n$ (b) $\bigcup_{n=1}^{\infty} Y_{2^n}$ (c) $\bigcup_{n=100}^{\infty} Y_n$ (d) $\bigcup_{n=1}^{\infty} Y_n$

34. Let $f : \mathbb{R} \setminus \{0\} \to \mathbb{R}$ be defined by $f(x) = x + \frac{1}{x^3}$. On which of the following interval (s) is *f* one-one?

(a) $(-\infty, -1)$ (b) $(0, 1)$ (c) $(0, 2)$ (d) $(0, \infty)$

35. Let *S* be a subset of $\mathbb{R}$ such that 2018 is an interior point of S. Which of the following is (are) TRUE?

(a) *S* contains an interval

(b) There is a sequence in *S* which does not converge to 2018

(c) There is an element $y \in S,\ y \neq 2018$ such that *y* is also an interior point of *S*

(d) There is a point $z \in S$, such that $|z - 2018| = 0.002018$

36. $\vec{F}(x, y, z) = (2x + 3yz)\hat{i} + (3xz + 2y)\hat{j} + (3xy + 2z)\hat{k}$ for $(x, y, z) \in \mathbb{R}^3$, then which among the following is (are) TRUE?

(a) $\nabla \times \vec{F} = \vec{0}$

(b) $\oint_C \vec{F} \cdot d\vec{r} = 0$ along any simple closed curve *C*

(c) There exists a scalar function $\phi : \mathbb{R}^3 \to \mathbb{R}$ such that $\nabla \cdot \vec{F} = \phi_{xx} + \phi_{yy} + \phi_{zz}$

(d) $\nabla \cdot \vec{F} = 0$

37. Let $m, n \in \mathbb{N}, m < n,\ P \in M_{n\times m}(\mathbb{R}),\ Q \in M_{m\times n}(\mathbb{R})$. Then which of the following is (are) NOT possible?

(a) $rank(PQ) = n$ (b) $rank(QP) = m$ (c) $rank(PQ) = m$

(d) $rank(QP) = \left\lceil \frac{m+n}{2} \right\rceil$, the smallest integer larger than or equal to $\frac{m+n}{2}$

38. Suppose $\alpha, \beta, \gamma \in \mathbb{R}$. Consider the following system of linear equations.

$x + y + z = \alpha, x + \beta y + z = \gamma, x + y + \alpha z = \beta$. If this system has at least one solution, then which of the following statements is (are) TRUE?

(a) If $\alpha = 1$ then $\gamma = 1$ (b) If $\beta = 1$ then $\gamma = \alpha$

(c) If $\beta \neq 1$ then $\alpha = 1$ (d) If $\gamma = 1$ then $\alpha = 1$

39. Suppose f, g, h are permutations of the set $\{\alpha, \beta, \gamma, \delta\}$, where

f interchanges α and β but fixes γ and δ

g interchanges β and γ but fixes α and δ

h interchanges γ and δ but fixes α and β

Which of the following permutations interchange (s) α and δ but fix(es) β and γ?

(a) $f \circ g \circ h \circ g \circ f$ (b) $g \circ h \circ f \circ h \circ g$ (c) $g \circ f \circ h \circ f \circ g$ (d) $h \circ g \circ f \circ g \circ h$

40. Which of the following subsets of $\mathbb{R}$ is (are) connected?

(a) $\{x \in \mathbb{R} \mid x^2 + x > 4\}$ (b) $\{x \in \mathbb{R} \mid x^2 + x < 4\}$

(c) $\{x \in \mathbb{R} \mid |x| < |x - 4|\}$ (d) $\{x \in \mathbb{R} \mid |x| > |x - 4|\}$

SECTION-C

[Numerical Answer Type (NAT)]

Q.41 – Q.50 carry ONE mark each.

41. Let $f(x)=\sum_{n=0}^{\infty}(-1)^n x(x-1)^n$ for $0<x<2$. Then the value of $f\left(\frac{\pi}{4}\right)$ is _________

42. Let $f(x,y)=\sqrt{x^3y}\sin\left(\frac{\pi}{2}e^{\left(\frac{y}{x}-1\right)}\right)+xy\cos\left(\frac{\pi}{3}e^{\left(\frac{x}{y}-1\right)}\right)$ for $(x,y)\in\mathbb{R}^2,\ \ x>0, y>0$.

Then $f_x(1,1)+f_y(1,1)=$ ___________

43. Let $\phi(x,y,z)=3y^2+3yz$ for $(x,y,z)\in\mathbb{R}^3$. Then the absolute value of the directional derivative of ϕ in the direction of the line $\frac{x-1}{2}=\frac{y-2}{-1}=\frac{z}{-2}$, at the point (1, –2, 1) is __________

44. Let $f:[0,\infty)\to[0,\infty)$ be continuous on $[0,\infty)$ and differentiable on $(0,\infty)$. If $f(x)=\int_0^x\sqrt{f(t)}\,dt$, then $f(6)$ = _________

45. Let $a_n=\frac{(1+(-1)^n)}{2^n}+\frac{(1+(-1)^{n-1})}{3^n}$. Then the radius of convergence of the power series $\sum_{n=1}^{\infty}a_nx^n$ about $x=0$ is __________

46. Let $f:\mathbb{R}^2\to\mathbb{R}$ be given by $f(x,y)=\begin{cases}\frac{x^2y(x-y)}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$.

Then $\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)-\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$ at the point (0, 0) is ___________

47. Let W_1 be the real vector space of all 5 × 2 matrices such that the sum of the entries in each row is zero. Let W_2 be the real vector space of all 5 × 2 matrices such that the sum of the entries in each column is zero. Then the dimension of the space $W_1\cap W_2$ is ____________

48. The coefficient of x^4 in the power series expansion of $e^{\sin x}$ about $x=0$ is ________ (correct upto three decimal places).

49. Let A_6 be the group of even permutations of 6 distinct symbols. Then the number of elements of order 6 in A_6 is __________

50. The order of the element (123)(246)(456) in the group S_6 is ____________

Q.51 – Q.60 carry TWO marks each.

51. If $y(x)=v(x)\sec x$ is the solution of $y''-(2\tan x)y'+5y=0,\ -\frac{\pi}{2}<x<\frac{\pi}{2}$, satisfying $y(0)=0$ and $y'(0)=\sqrt{6}$, then $v\left(\frac{\pi}{6\sqrt{6}}\right)$ is _____________ (correct upto two decimal places).

52. Suppose x, y, z are positive real numbers such that $x+2y+3z=1$. If M is the maximum value of xyz^2, then the value of $\frac{1}{M}$ is ___________

53. If $\alpha = \int_{\pi/6}^{\pi/3} \frac{\sin t + \cos t}{\sqrt{\sin 2t}}\,dt$, then the value of $\left(2\sin\frac{\alpha}{2}+1\right)^2$ is _____________

54. Suppose $Q \in M_{3\times3}(\mathbb{R})$ is a matrix of rank 2. Let $T : M_{3\times3}(\mathbb{R}) \to M_{3\times3}(\mathbb{R})$ be the linear transformation defined by $T(P) = QP$. Then the rank of T is __________

55. Let $a_k = (-1)^{k-1}, s_n = a_1 + a_2 + ... + a_n$ and $\sigma_n = (s_1 + s_2 + ... + s_n)/n$, where $k, n \in \mathbb{N}$.

Then $\lim_{n\to\infty} \sigma_n$ is ____________ (correct upto one decimal place)

56. The value of the integral $\int_0^1 \int_x^1 y^4 e^{xy^2}\,dy\,dx$ is ____________ (correct upto three decimal places)

57. The area of the parametrized surface

$S = \{((2+\cos u)\cos v, (2+\cos u)\sin v, \sin u) \in \mathbb{R}^3 \mid 0 \le u \le \frac{\pi}{2}, 0 \le v \le \frac{\pi}{2}\}$ is ____________

(correct upto two decimal places)

58. Let $f : \mathbb{R} \to \mathbb{R}$ be such that f'' is continuous on $\mathbb{R}$ and $f(0) = 1$, $f'(0) = 0$ and $f''(0) = -1$.

Then $\lim_{x\to\infty}\left(f\left(\sqrt{\frac{2}{x}}\right)\right)^x$ is ____________ (correct upto three decimal places)

59. If $x(t)$ is the solution to the differential equation $\frac{dx}{dt} = x^2t^3 + xt$, for $t > 0$, satisfying $x(0) = 1$, then the value of $x(\sqrt{2})$ is ____________ (correct upto two decimal places)

60. If the volume of the solid in $\mathbb{R}^3$ bounded by the surfaces $x=-1,\ x=1,\ y=-1,\ y=1,\ z=2,\ y^2+z^2=2$ is $\alpha - \pi$, then $\alpha =$ _______________

***** END *****