PAPER : IIT-JAM

MATHEMATICS MA–2019

SECTION-A

[Multiple Choice Questions (MCQ)]

Q.1 – Q.10 carry ONE mark each.

1. The equation of the tangent plane to the surface $x^2z+\sqrt{8-x^2-y^4}=6$ at the point (2, 0, 1) is

(a) $2x+z=5$ (b) $3x+4z=10$ (c) $3x-z=10$ (d) $7x-4z=10$

2. Let $a_1=b_1=0$, and for each $n\geq 2$, let a_n and b_n be real numbers given by

$$a_n=\sum_{m=2}^{n}\frac{(-1)^m m}{(\log(m))^m} \text{ and } b_n=\sum_{m=2}^{n}\frac{1}{(\log(m))^m}.$$

Then which one of the following is TRUE about the sequences $\{a_n\}$ and $\{b_n\}$?

(a) Both $\{a_n\}$ and $\{b_n\}$ are divergent

(b) $\{a_n\}$ is convergent and $\{b_n\}$ is divergent

(c) $\{a_n\}$ is divergent and $\{b_n\}$ is convergent

(d) Both $\{a_n\}$ and $\{b_n\}$ are convergent

3. If $y(x)=\lambda e^{2x}+e^{\beta x},\beta\neq 2$, is a solution of the differential equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}-6y=0$, satisfying $\frac{dy}{dx}(0)=5$, then $y(0)$ is equal to

(a) 1 (b) 4 (c) 5 (d) 9

4. Let $T\in M_{m\times n}(\mathbb{R})$. Let V be the subspace of $M_{n\times p}(\mathbb{R})$ defined by $V=\{X\in M_{n\times p}(\mathbb{R}):TX=0\}$. Then the dimension of V is

(a) pn - rank(T) (b) mn - p rank(T) (c) $p(m$ - rank (T)) (d) $p(n$ - rank(T))

5. If $x^h y^k$ is an integrating factor of the differential equation $y(1+xy)dx+x(1-xy)dy=0$. Then the ordered pair (h, k) is equal to

(a) (-2, -2) (b) (-2, -1) (c) (-1, -2) (d) (-1, -1)

6. Let S be the set of all limit points of the set $\left\{\frac{n}{\sqrt{2}}+\frac{\sqrt{2}}{n}:n\in\mathbb{N}\right\}$. Let $\mathbb{Q}_+$ be the set of all positive rational numbers. Then

(a) $\mathbb{Q}_+\subseteq S$ (b) $S\subseteq\mathbb{Q}_+$ (c) $S\cap(\mathbb{R}\setminus\mathbb{Q}_+)\neq 0$ (d) $S\cap\mathbb{Q}_+\neq 0$

7. Let $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ be sequences of positive real number such that $na_n<b_n<n^2a_n$ for all $n\geq 2$. If the radius of convergence of the power series $\sum_{n=0}^{\infty}a_nx^n$ is 4, then the power series $\sum_{n=0}^{\infty}b_nx^n$.

(a) converges for all x with $|x|<2$

(b) converges for all x with $|x|>2$

(c) does not converge for any x with $|x|>2$

(d) does not converge for any x with $|x|<2$

8. The area of the surface generated by rotating the curve $x = y^3, 0 \le y \le 1$, about the y-axis, is

(a) $\frac{\pi}{27}10^{3/2}$ (b) $\frac{4\pi}{3}(10^{3/2}-1)$ (c) $\frac{\pi}{27}(10^{3/2}-1)$ (d) $\frac{4\pi}{3}10^{3/2}$

9. The value of the integral $\int_{y=0}^{1}\int_{x=0}^{1-y^2} y\sin(\pi(1-x)^2)\,dx\,dy$ is

(a) $\frac{1}{2\pi}$ (b) 2π (c) $\frac{\pi}{2}$ (d) $\frac{2}{\pi}$

10. Let $g : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. Define $f : \mathbb{R}^3 \to \mathbb{R}$ by $f(x,y,z) = g(x^2+y^2-2z^2)$.

Then $\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2}$ is equal to

(a) $4(x^2+y^2-4z^2)g''(x^2+y^2-2z^2)$

(b) $4(x^2+y^2+4z^2)g''(x^2+y^2-2z^2)$

(c) $4(x^2+y^2-2z^2)g''(x^2+y^2-2z^2)$

(d) $4(x^2+y^2+4z^2)g''(x^2+y^2-2z^2)+8g'(x^2+y^2-2z^2)$

Q.11 – Q.30 carry TWO marks each.

11. For $\beta \in \mathbb{R}$, define $f(x,y) = \begin{cases} \frac{x^2|x|^\beta y}{x^4+y^2}, & x \ne 0 \\ 0, & x = 0 \end{cases}$. Then at (0, 0) the function f is

(a) continuous for $\beta = 0$ (b) continuous for $\beta > 0$

(c) not differentiable for any β (d) continuous for $\beta < 0$

12. The set $\left\{\frac{x}{1+x} : -1 < x < 1\right\}$, as a subset of $\mathbb{R}$, is

(a) connected and compact (b) connected but not compact

(c) not connected but compact (d) neither connected nor compact

13. The set $\left\{\frac{1}{m}+\frac{1}{n} : m,n \in \mathbb{N}\right\} \cup \{0\}$, as a subset of $\mathbb{R}$ is

(a) compact and open (b) compact but not open

(c) not compact but open (d) neither compact nor open

14. Let $f(x) = (\ln x)^2, x > 0$. Then

(a) $\lim_{x\to\infty}\frac{f(x)}{x}$ does not exist (b) $\lim_{x\to\infty} f'(x) = 2$

(c) $\lim_{x\to\infty}(f(x+1)-f(x)) = 0$ (d) $\lim_{x\to\infty}(f(x+1)-f(x))$ does not exist

15. Which one of the following series is divergent ?

(a) $\sum_{n=1}^{\infty}\frac{1}{n}\sin^2\frac{1}{n}$ (b) $\sum_{n=1}^{\infty}\frac{1}{n}\log n$ (c) $\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\frac{1}{n}$ (d) $\sum_{n=1}^{\infty}\frac{1}{n}\tan\frac{1}{n}$

16. The function $f(x,y)=x^3+2xy+y^3$ has a saddle point at

(a) $(0,0)$ (b) $\left(-\frac{2}{3},-\frac{2}{3}\right)$ (c) $\left(-\frac{3}{2},-\frac{3}{2}\right)$ (d) (-1, -1)

17. For which one of the following values of *k*, the equation $2x^3+3x^2-12x-k=0$ has three distinct real roots ?

(a) 16 (b) 20 (c) 26 (d) 31

18. For $-1<x<1$, the sum of the power series $1+\sum_{n=2}^{\infty}(-1)^{n-1}n^2x^{n-1}$ is

(a) $\frac{1-x}{(1+x)^3}$ (b) $\frac{1+x^2}{(1+x)^4}$ (c) $\frac{1-x}{(1+x)^2}$ (d) $\frac{1+x^2}{(1+x)^3}$

19. Let C be the circle $(x-1)^2+y^2=1$, oriented counter clockwise. Then the value of the line integral

$\oint_C -\frac{4}{3}xy^3dx+x^4dy$ is

(a) 6π (b) 8π (c) 12π (d) 14π

20. Let *P* be a 4 × 4 matrix with entries from the set of rational numbers. If $\sqrt{2}+i$, with $i=\sqrt{-1}$ is a root of the characteristic polynomial of *P* and *I* is the 4 × 4 identity matrix, then

(a) $P^4=4P^2+9I$ (b) $P^4=4P^2-9I$ (c) $P^4=2P^2-9I$ (d) $P^4=2P^2+9I$

21. The set of eigenvalues of which one of the following matrices is NOT equal to the set of eigenvalues of $\begin{pmatrix}1&2\\4&3\end{pmatrix}$?

(a) $\begin{pmatrix}1&4\\2&3\end{pmatrix}$ (b) $\begin{pmatrix}3&2\\4&1\end{pmatrix}$ (c) $\begin{pmatrix}3&4\\2&1\end{pmatrix}$ (d) $\begin{pmatrix}2&3\\1&4\end{pmatrix}$

22. Let $\{a_n\}$ be a sequence of positive real numbers. The series $\sum_{n=1}^{\infty}a_n$ converges if the series

(a) $\sum_{n=1}^{\infty}a_n^2$ converges (b) $\sum_{n=1}^{\infty}\frac{a_n}{2^n}$ converges

(c) $\sum_{n=1}^{\infty}\frac{a_{n+1}}{a_n}$ converges (d) $\sum_{n=1}^{\infty}\frac{a_n}{a_{n+1}}$ converges

23. Let $\{a_n\}$ be a sequence of positive real numbers such that $a_1=1, a_{n+1}^2-2a_na_{n+1}-a_n=0$ for all $n\geq 1$.

Then the sum of the series $\sum_{n=1}^{\infty}\frac{a_n}{3^n}$ lies in the interval

(a) $(1,2]$ (b) $(2,3]$ (c) $(3,4]$ (d) $(4,5]$

24. The tangent line to the curve of intersection of the surface $x^2+y^2-z=0$ and the plane $x+z=3$ at the point (1, 1, 2) passes through

(a) (-1, -2, 4) (b) (-1, 4, 4) (c) (3, 4, 4) (d) (-1, 4, 0)

25. The area of the part of the surface of the paraboloid $x^2+y^2+z=8$ lying inside the cylinder $x^2+y^2=4$ is

(a) $\frac{\pi}{2}(17^{3/2}-1)$ (b) $\pi(17^{3/2}-1)$ (c) $\frac{\pi}{6}(17^{3/2}-1)$ (d) $\frac{\pi}{3}(17^{3/2}-1)$

26. Let $\vec{F}(x,y,z)=2y\,\hat{i}+x^2\hat{j}+xy\hat{k}$ and C be the curve of intersection of the plane $x+y+z=1$ and the cylinder $x^2+y^2=1$. Then the value of $\left|\oint_C \vec{F}\cdot d\vec{r}\right|$ is

(a) π (b) $\frac{3\pi}{2}$ (c) 2π (d) 3π

27. Let S be the family of orthogonal trajectories of the family of curves $2x^2+y^2=k$, for $k\in\mathbb{R}$ and $k>0$. If $C\in S$ and C passes through the point (1, 2), then C also passes through

(a) $(4,-\sqrt{2})$ (b) (2, -4) (c) $(2,2\sqrt{2})$ (d) $(4,2\sqrt{2})$

28. Let H and K be subgroups of $\mathbb{Z}_{144}$. If the order of H is 24 and the order of K is 36, then the order of the subgroup $H\cap K$ is

(a) 3 (b) 4 (c) 6 (d) 12

29. Let $f:\mathbb{R}\to\mathbb{R}$ be a differentiable function such that $f'(x)>f(x)$ for all $x\in\mathbb{R}$ and $f(0)=1$. Then $f(1)$ lies in the interval

(a) $(0,e^{-1})$ (b) $(e^{-1},\sqrt{e})$ (c) $(\sqrt{e},e)$ (d) (e,∞)

30. Let $x, x+e^x$ and $1+x+e^x$ be solutions of a linear second order ordinary differential equation with constant coefficients. If $y(x)$ is the solution of the same equation satisfying $y(0)=3$ and $y'(0)=4$, then $y(1)$ is equal to

(a) $e+1$ (b) $2e+3$ (c) $3e+2$ (d) $3e+1$

SECTION-B

[Multiple Select Questions (MSQ)]

Q.31 – Q.40 carry TWO marks each.

31. Let $\vec{F}$ and $\vec{G}$ be differentiable vector fields and let g be a differentiable scalar function. Then

(a) $\nabla\cdot(\vec{F}\times\vec{G})=\vec{G}\cdot\nabla\times\vec{F}-\vec{F}\cdot\nabla\times\vec{G}$

(b) $\nabla\cdot(\vec{F}\times\vec{G})=\vec{G}\cdot\nabla\times\vec{F}+\vec{F}\cdot\nabla\times\vec{G}$

(c) $\nabla\cdot(g\vec{F})=g\nabla\cdot\vec{F}-\nabla g\cdot\vec{F}$

(d) $\nabla\cdot(g\vec{F})=g\nabla\cdot\vec{F}+\nabla g\cdot\vec{F}$

32. Let S and T be linear transformations from a finite dimensional vector space V to itself such that $S(T(v))=0$ for all $v\in V$. Then

(a) rank(T) $\geq$ nullity(S)

(b) rank(S) $\geq$ nullity(S)

(c) rank(T) $\leq$ nullity(S)

(d) rank(S) $\leq$ nullity(T)

33. Let $f(x,y)=\begin{cases}\dfrac{|x|}{|x|+|y|}\sqrt{x^4+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$. Then at (0, 0).

(a) f is continuous

(b) $\frac{\partial f}{\partial x}=0$ and $\frac{\partial f}{\partial y}$ does not exist

(c) $\frac{\partial f}{\partial x}$ does not exist and $\frac{\partial f}{\partial y}=0$

(d) $\frac{\partial f}{\partial x}=0$ and $\frac{\partial f}{\partial y}=0$

34. Let $\{a_n\}$ be the sequence of real numbers such that $a_1 = 1$ and $a_{n+1} = a_n + a_n^2$ for all $n \geq 1$. Then

(a) $a_4 = a_1(1+a_1)(1+a_2)(1+a_3)$

(b) $\lim_{n\to\infty} \frac{1}{a_n} = 0$

(c) $\lim_{n\to\infty} \frac{1}{a_n} = 1$

(d) $\lim_{n\to\infty} a_n = 0$

35. Let $f:\left(0,\frac{\pi}{2}\right)\to\mathbb{R}$ be given by $f(x) = (\sin x)^{\pi} - \pi\sin x + \pi$. Then which of the following statements is/are TRUE?

(a) f is an increasing function

(b) f is a decreasing function

(c) $f(x) > 0$ for all $x \in \left(0,\frac{\pi}{2}\right)$

(d) $f(x) < 0$ for some $x \in \left(0,\frac{\pi}{2}\right)$

36. Consider the intervals $S = (0,2]$ and $T = [1,3)$. Let S° and T° be the sets of interior points of S and T, respectively. Then the set of interior points of $S\setminus T$ is equal to

(a) $S\setminus T^{\circ}$

(b) $S\setminus T$

(c) $S^{\circ}\setminus T^{\circ}$

(d) $S^{\circ}\setminus T$

37. Let $\{a_n\}$ be the sequence given by $a_n = \max\left\{\sin\left(\frac{n\pi}{3}\right),\cos\left(\frac{n\pi}{3}\right)\right\}, n \geq 1$. Then which of the following statements is/are TRUE about the subsequences $\{a_{6n-1}\}$ and $\{a_{6n+4}\}$?

(a) Both the subsequences are convergent

(b) Only one of the subsequences is convergent

(c) $\{a_{6n-1}\}$ converges to $-\frac{1}{2}$

(d) $\{a_{6n+4}\}$ converges to $\frac{1}{2}$

38. Let $f(x) = \cos(|\pi - x|) + (x-\pi)\sin|x|$ and $g(x) = x^2$ for $x \in \mathbb{R}$. If $h(x) = f(g(x))$, then

(a) h is not differentiable at $x = 0$

(b) $h'(\sqrt{\pi}) = 0$

(c) $h''(x) = 0$ has a solution in $(-\pi,\pi)$

(d) there exists $x_0 \in (-\pi,\pi)$ such that $h(x_0) = x_0$

39. Let G be a non cyclic group of order 4. Consider the statements I and II.

I. There is NO injective (one-one) homomorphism from G to $\mathbb{Z}_8$.

II. There is NO surjective (onto) homomorphism from $\mathbb{Z}_8$ to G

Then

(a) I is true

(b) I is false

(c) II is true

(d) II is false

40. Let G be non abelian group, $y \in G$ and let the maps f, g, h from G to itself be defined by $f(x) = yxy^{-1}, g(x) = x^{-1}$ and $h = g\circ g$. Then

(a) g and h are homomorphisms and f is not a homomorphism

(b) h is a homomorphism and g is not a homomorphism

(c) f is a homomorphism and g is not a homomorphism

(d) f, g and h are homomorphism

SECTION-C

[Numerical Answer Type (NAT)]

Q.1 – Q.10 carry ONE mark each.

41. If $g(x)=\int_{x(x-2)}^{4x-5} f(t)\,dt$, where $f(x)=\sqrt{1+3x^4}$ for , $x\in\mathbb{R}$ then $g'(1)=$___________.

42. Let x be the 100-cycle (1, 2, 3 ... 100) and let y be the transposition (49, 50) in the permutation group S_{100}. Then the order of xy is___________ .

43. The volume of the solid bounded by the surfaces $x=1-y^2$ and $x=y^2-1$, and the planes $z=0$ and $z=2$ (round off to 2 decimal places) is___________.

44. The coefficient of $\left(x-\frac{\pi}{2}\right)$ in the Taylor series expansion of the function $f(x)=\begin{cases}\dfrac{4(1-\sin x)}{2x-x}, & x\neq\dfrac{\pi}{2}\\ 0, & x=\dfrac{\pi}{2}\end{cases}$,

about $x=\frac{\pi}{2}$ is ___________.

45. Let $f(x,y)=\begin{cases}\dfrac{x^3+y^3}{x^2-y^2}, & x^2-y^2\neq 0\\ 0, & x^2-y^2=0\end{cases}$. Then the directional derivative of f at (0, 0) in the direction of $\frac{4}{5}\hat{i}+\frac{3}{5}\hat{j}$ is___________.

46. Consider the following system of three linear equations in four unknowns x_1, x_2, x_3 and x_4

$$\begin{aligned} x_1+x_2+x_3+x_4 &= 4\\ x_1+2x_2+3x_3+4x_4 &= 5\\ x_1+3x_2+5x_3+kx_4 &= 5\end{aligned}$$

. If the system has no solutions, then $k=$___________.

47. The value of the integral $\int_{-1}^{1}\int_{-1}^{1}|x+y|\,dx\,dy$ (round off to 2 decimal places) is___________ .

48. Let W_1 and W_2 be subspaces of the real vector space $\mathbb{R}^{100}$ defined by

$W_1=\{(x_1,x_2,.....,x_{100}):x_i=0$ if i is divisible by 4$\}$

$W_2=\{(x_1,x_2,.....,x_{100}):x_i=0$ if i is divisible by 5$\}$.

Then the dimension of $W_1\cap W_2$ is___________ .

49. Let $f:[0,1]\to\mathbb{R}$ be given by $f(x)=\dfrac{\left(1+x^{\frac{1}{3}}\right)^3+\left(1-x^{\frac{1}{3}}\right)^3}{8(1+x)}$.

Then $\max\{f(x):x\in[0,1]\}-\min\{f(x):x\in[0,1]\}$ is ___________.

50. Let $\vec{F}(x,y) = -y\hat{i} + x\hat{j}$ and let C be the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ oriented counter clockwise. Then the values of $\oint_C \vec{F} \cdot d\vec{r}$ (round off to 2 decimal places) is ___________ .

Q.51 – Q.60 carry TWO marks each.

51. The number of critical points of the function $f(x,y) = (x^2 + 3y^2)e^{-(x^2+y^2)}$ is ___________ .

52. The volume of the solid of revolution of the loop of the curve $y^2 = x^4(x+2)$ about the x-axis (round off to 2 decimal places) is ___________ .

53. If $y(x)$ is the solution of th e initial value problem $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0, y(0) = 2, \frac{dy}{dx}(0) = 0$. Then $y(\ln 2)$ is (round off to 2 decimal places) equal to___________ .

54. Let $G = \{n \in \mathbb{N} : n \le 55, \gcd(n,55) = 1\}$ be the group under multiplication modulo 55. Let $x \in G$ be such that $x^2 = 26$ and $x > 30$. Then x is equal to ___________ .

55. Let M and N be any two 4×4 matrices with integer entries satisfying

$$MN = 2\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Then the maximum value of det(M) + det(N) is ___________ .

56. Let M be a 3×3 matrix with real entries such that $M^2 = M + 2I$, where I denotes the 3×3 identity matrix. If α, β and γ are eigenvalues of M such that $\alpha\beta\gamma = -4$, then $\alpha + \beta + \gamma$ is equal to ___________ .

57. If $\begin{pmatrix} 2 \\ y \\ z \end{pmatrix}; y, z \in \mathbb{R}$ is an eigenvector corresponding to real eigenvalue of the matrix $\begin{pmatrix} 0 & 0 & 2 \\ 1 & 0 & -4 \\ 0 & 1 & 3 \end{pmatrix}$ then $z - y$ is equal to ___________ .

58. The greatest lower bound to the set $\{(e^n + 2^n)^{\frac{1}{n}} : n \in \mathbb{N}\}$, (round off to 2 decimal places) is__________ .

59. The number of elements in the set $\{x \in S_3 : x^4 = e\}$, where e is the identity element of the permutation group S_3, is___________ .

60. Let $y(x) = xv(x)$ be a solution of the differential equation $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0$. If $v(0) = 0$ and $v(1) = 1$, then $v(-2)$ is equal to ___________ .

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