PAPER : IIT-JAM

MATHEMATICS MA–2020

SECTION-A

[Multiple Choice Questions (MCQ)]

Q.1 – Q.10 carry ONE mark each.

1. Let $f(x) = 2x^3 - 9x^2 + 7$. Which of the following is true ?

(a) f is one-one in the interval [-1,1]
(b) f is one-one in the interval [2,4]
(c) f is not one-one in the interval [-4,0]
(d) f is not one-one in the interval [0,4]

2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by $T(x, y) = (-x, y)$. Then

(a) $T^{2k} = T$ for all $k \geq 1$
(b) $T^{2k+1} = -T$ for all $k \geq 1$
(c) The range of T^2 is a proper subspace of the range of T
(d) The range of T^2 is equal to the range of T

3. If $u = x^3$ and $v = y^2$ tansfrom the differential equation $3x^5 dx - y\left(y^2 - x^3\right)dy = 0$ to $\dfrac{dv}{du} = \dfrac{\alpha u}{2(u - v)}$, then α is

(a) 4
(b) 2
(c) –2
(d) –4

4. Which of the following is False?

(a) $\lim\limits_{x\to\infty} \dfrac{x}{e^x} = 0$
(b) $\lim\limits_{x\to 0^+} \dfrac{1}{xe^{1/x}} = 0$
(c) $\lim\limits_{x\to 0^+} \dfrac{\sin x}{1+2x} = 0$
(d) $\lim\limits_{x\to 0^+} \dfrac{\cos x}{1+2x} = 0$

5. The radius of convergence of the power series $\sum\limits_{n=1}^{\infty} \left(\dfrac{n+2}{n}\right)^{n^2} x^n$ is

(a) e^2
(b) $\dfrac{1}{\sqrt{e}}$
(c) $\dfrac{1}{e}$
(d) $\dfrac{1}{e^2}$

6. Let $g : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. If $f(x, y) = g(y) + xg'(y)$, then

(a) $\dfrac{\partial f}{\partial x} + y\dfrac{\partial^2 f}{\partial x dy} = \dfrac{\partial f}{\partial y}$
(b) $\dfrac{\partial f}{\partial y} + y\dfrac{\partial^2 f}{\partial x dy} = \dfrac{\partial f}{\partial x}$
(c) $\dfrac{\partial f}{\partial x} + x\dfrac{\partial^2 f}{\partial x dy} = \dfrac{\partial f}{\partial y}$
(d) $\dfrac{\partial f}{\partial y} + x\dfrac{\partial^2 f}{\partial x dy} = \dfrac{\partial f}{\partial x}$

7. Let $s_n = 1 + \dfrac{(-1)^n}{n}, n \in \mathbb{N}$. Then the sequence $\{s_n\}$ is

(a) monotonically increasing and is convergent to 1
(b) monotonically decreasing and is convergent to 1
(c) neither monotonically increasing nor monotonically decreasing but is convergent to 1
(d) divergent

8. Consider the following group under matrix multiplication

$$H = \left\{ \begin{bmatrix} 1 & p & q \\ 0 & 1 & r \\ 0 & 0 & 1 \end{bmatrix} : p, qr \in \mathbb{R} \right\}$$

Then the center of the group is isomorphic to

(a) $(\mathbb{R} \setminus \{0\}, \times)$
(b) $(\mathbb{R}, +)$
(c) $(\mathbb{R}^2, +)$
(d) $(\mathbb{R}, +) \times (\mathbb{R} \setminus \{0\}, \times)$

9. If the directional derivative of the function $z = y^2 e^{2x}$ at $(2, -1)$ along the unit vactor $\vec{b} = \alpha\hat{i} + \beta\hat{j}$ is zero, then $|\alpha + \beta|$ equals.

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) $2\sqrt{2}$

10. If the equation of the tangent plane to the surface $z = 16 - x^2 - y^2$ at the point $P(1,3,6)$ is $ax + by + cz + d = 0$. Then the value of $|d|$ is

(a) 16 (b) 26 (c) 36 (d) 46

11. Let M be a 4×3 real matrix and let $\{e_1, e_2, e_3\}$ be the standard basis of $\mathbb{R}^3$. which of the following is true ?

(a) If rank $(M) = 1$, then $\{Me_1, Me_2\}$ is a linearly independent set

(b) If rank $(M) = 2$, then $\{Me_1, Me_2\}$ is a linearly independent set

(c) If rank $(M) = 2$, then $\{Me_1, Me_3\}$ is a linearly independent set

(d) If rank $(M) = 3$, then $\{Me_1, Me_3\}$ is a linearly independent set

12. Let $S^1 = \{z \in \mathbb{C} : |z| = 1\}$ be the circle group under multiplication and $i = \sqrt{-1}$. Then the set $\{\theta \in \mathbb{R} : \langle e^{i2\pi\theta} \rangle \text{ is infinite}\}$ is

(a) empty (b) non- empty and finite

(c) countably infinite (d) uncountable

13. Define $s_1 = \alpha > 0$ and $s_{n+1} = \sqrt{\frac{1 + s_n^2}{1 + \alpha}}, n \geq 1$. Which of the following is true ?

(a) If $s_n^2 < \frac{1}{\alpha}$, then $\{s_n\}$ is monotonically increasing and $\lim_{n\to\infty} s_n = \frac{1}{\sqrt{\alpha}}$

(b) If $s_n^2 < \frac{1}{\alpha}$, then $\{s_n\}$ is monotonically decreasing and $\lim_{n\to\infty} s_n = \frac{1}{\alpha}$

(c) If $s_n^2 > \frac{1}{\alpha}$, then $\{s_n\}$ is monotonically increasing and $\lim_{n\to\infty} s_n = \frac{1}{\sqrt{\alpha}}$

(d) If $s_n^2 > \frac{1}{\alpha}$, then $\{s_n\}$ is monotonically decreasing and $\lim_{n\to\infty} s_n = \frac{1}{\alpha}$

14. Let M be a real 6×6 matrix. Let 2 and –1 be two eigenvalues of M. If $M^5 = aI + bM$, where $a, b \in \mathbb{R}$, then

(a) $a = 10, b = 11$ (b) $a = -11, b = 10$ (c) $a = -10, b = 11$ (d) $a = -10, b = -11$

15. Let $f : [0,1] \to \mathbb{R}$ be a continuous function such that $f\left(\frac{1}{2}\right) = -\frac{1}{2}$ and

$|f(x) - f(y) - (x - y)| \leq \sin\left(|x - y|^2\right)$ for all $x, y \in [0,1]$. Then $\int_0^1 f(x)dx$ is

(a) $-\frac{1}{2}$ (b) $-\frac{1}{4}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

16. Let $f(x,y)=\begin{cases} x^2\sin\frac{1}{x}+y^2\sin\frac{1}{y}, & xy\neq 0 \\ x^2\sin\frac{1}{x}, & x\neq 0, y=0 \\ y^2\sin\frac{1}{y}, & y\neq 0, x=0 \\ 0, & x=y=0 \end{cases}$

Which of the following is true at (0, 0) ?

(a) f is not continuous

(b) $\frac{\partial f}{\partial x}$ is continuous but $\frac{\partial f}{\partial y}$ is not continuous

(c) f is not differentiable

(d) f is differentiable but both $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are not continuous

17. Suppose that S is the sum of a convergent series $\sum_{n=1}^{\infty} a_n$. Define $t_n = a_n + a_{n+1} + a_{n+2}$. Then the series $\sum_{n=1}^{\infty} t_n$

(a) diverges
(b) converges to $3S - a_1 - a_2$
(c) converges to $3S - a_1 - 2a_2$
(d) converges to $3S - 2a_1 - a_2$

18. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, $x,y,z\in\mathbb{R}$. Which of the following is False ?

(a) $\nabla(\vec{a}\cdot\vec{r})=\vec{a}$
(b) $\nabla(\vec{a}\times\vec{r})=0$
(c) $\nabla\times(\vec{a}\times\vec{r})=\vec{a}$
(d) $\nabla\cdot((\vec{a}\cdot\vec{r})\vec{r})=4(\vec{a}\cdot\vec{r})$

19. Let $F=\{\omega\in\mathbb{C}:\omega^{2020}=1\}$. Consider the groups

$G=\left\{\begin{pmatrix}\omega & z\\ 0 & 1\end{pmatrix}:\omega\in F, z\in\mathbb{C}\right\}$ and $H=\left\{\begin{pmatrix}1 & z\\ 0 & 1\end{pmatrix}:z\in\mathbb{C}\right\}$

under matrix multiplication. Then the number of cosets of H in G is

(a) 1010 (b) 2019 (c) 2020 (d) infinite

20. Let $f(x,y,z)=x^3+y^3+z^3-3xyz$. A point at which the gradient of the function f is equal to zero is

(a) (–1,1,–1) (b) (–1,–1,–1) (c) (–1,1,1) (d) (1,–1,1)

21. Let $a\in\mathbb{R}$. If $f(x)=\begin{cases}(x+a)^2 & ,x\leq 0\\ (x+a)^3 & ,x>0\end{cases}$

then

(a) $\frac{d^2f}{dx^2}$ does not exist at $x=0$ for any value of a
(b) $\frac{d^2f}{dx^2}$ exists at $x=0$ for exactly one value of a
(c) $\frac{d^2f}{dx^2}$ exists at $x=0$ for exactly two values of a
(d) $\frac{d^2f}{dx^2}$ exists at $x=0$ for infinitely many values of a

22. A solution of the differential equation $2x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}-y=0, x>0$ that passes through the point (1, 1) is

(a) $y=\frac{1}{x}$ (b) $y=\frac{1}{x^2}$ (c) $y=\frac{1}{\sqrt{x}}$ (d) $y=\frac{1}{x^{3/2}}$

23. Consider the differential equation $L[y]=\left(y-y^2\right)dx+xdy=0$. The function $f(x,y)$ is said to be an integrating factor of the equation if $f(x,y)L[y]=0$ becomes exact. If $f(x,y)=\frac{1}{x^2y^2}$, then

(a) f is an integrating factor and $y=1-kxy, k\in\mathbb{R}$ is NOT its general solution

(b) f is an integrating factor and $y=-1+kxy, k\in\mathbb{R}$ is its general solution

(c) f is an integrating factor and $y=-1+kxy, k\in\mathbb{R}$ is NOT its general solution

(d) f is NOT an integrating factor and $y=1+kxy, k\in\mathbb{R}$ is its general solution

24. Let M be an $n\times n(n\geq 2)$ non-zero real matrix with $M^2=0$ and let $\alpha\in\mathbb{R}\setminus\{0\}$. Then

(a) α is only eigenvalue of $(M+\alpha I)$ and $(M-\alpha I)$

(b) α is only eigenvalue of $(M+\alpha I)$ and $(\alpha I-M)$

(c) $-\alpha$ is the only eigenvalue of $(M+\alpha I)$ and $(M-\alpha I)$

(d) $-\alpha$ is only eigenvalue of $(M+\alpha I)$ and $(\alpha I-M)$

25. Let $\{a_n\}$ be a sequence of positive real numbers. Suppose that $l=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}$.which of the following is true ?

(a) If $l=1$, then $\lim_{n\to\infty}a_n=1$ (b) If $l=1$, then $\lim_{n\to\infty}a_n=0$

(c) If $l<1$, then $\lim_{n\to\infty}a_n=1$ (d) If $l<1$, then $\lim_{n\to\infty}a_n=0$

26. Let $D=\left\{(x,y)\in\mathbb{R}^2:|x|+|y|\leq 1\right\}$ and $f:D\to\mathbb{R}$ be a non- constant continuous function. Which of the following is TRUE ?

(a) The range of f is unbounded

(b) The range of f is a union of open intervals

(c) The range of f is a closed interval

(d) the range of f is a union of at least two disjoint closed intervals

27. The area bounded by the curves $x^2+y^2=2x$ and $x^2+y^2=4x$, and the straight lines $y=x$ and $y=0$ is

(a) $3\left(\frac{\pi}{2}+\frac{1}{4}\right)$ (b) $3\left(\frac{\pi}{4}+\frac{1}{2}\right)$ (c) $2\left(\frac{\pi}{4}+\frac{1}{3}\right)$ (d) $2\left(\frac{\pi}{3}+\frac{1}{4}\right)$

28. Let S be the surface of the portion of the sphere with centre at the origin and radius 4, above the xy-plane. Let $\vec{F}=y\hat{i}-x\hat{j}+yx^3\hat{k}$. if $\hat{n}$ is the unit outward normal to S, Then $\iint_S\left(\nabla\times\vec{F}\right)\cdot\hat{n}dS$ equals

(a) -32π (b) -16π (c) 16π (d) 32π

29. The value of the triple integral $\iiint_V\left(x^2y+1\right)dxdydz$, where V is the region given by $x^2+y^2\leq 1, 0\leq z\leq 2$ is

(a) π (b) 2π (c) 3π (d) 4π

30. Let S be the part of the cone $z^2=x^2+y^2$ between the planes z = 0 and z = 1. Then the value of the surface integral $\iint_S\left(x^2+y^2\right)dS$ is

(a) π (b) $\frac{\pi}{\sqrt{2}}$ (c) $\frac{\pi}{\sqrt{3}}$ (d) $\frac{\pi}{2}$

SECTION-B

[Multiple Select Questions (MSQ)]

Q.31 – Q.40 carry TWO marks each.

31. Let $a = \lim_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+..+\frac{(n-1)}{n^2}\right)$ and $b = \lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+..+\frac{1}{n+n}\right)$ which of the following is/are true?

(a) $a > b$ (b) $a < b$ (c) $ab = \ln\sqrt{2}$ (d) $\frac{a}{b} = \ln\sqrt{2}$

32. Let $L[y] = x^2\frac{d^2y}{dx^2} + px\frac{dy}{dx} + qy$, where p,q are real constants Let $y_1(x)$ and $y_2(x)$ be two solution of $L[y]=0,\ x>0$ that satisfy $y_1(x_0)=1, y_1'(x_0)=0$, $y_2(x_0)=0$ and $y_2'(x_0)=1$ for some $x_0>0$. Then,

(a) $y_1(x)$ is not a constant multiple of $y_2(x)$

(b) $y_1(x)$ is constant multiple of $y_2(x)$

(c) $1, \ln x$ are solutions of $L[y]=0$ when $p=1, q=0$

(d) $x, \ln x$ are solutions of $L[y]=0$ when $p+q\neq 0$

33. Cosider the following system of linear equations $x+y+5z=3,\ x+2y+mz=5$ and $x+2y+4z=k$. The system is consistent if

(a) $m\neq 4$ (b) $k\neq 5$ (c) $m=4$ (d) $k=5$

34. Let $a,b\in\mathbb{R}$ and $a<b$. Which of the following statement(s) is/are true ?

(a) There exists a continuous function $f:[a,b]\to(a,b)$ such that f is one-one

(b) There exists a continuous function $f:[a,b]\to(a,b)$ such that f is onto

(c) There exists a continuous function $f:(a,b)\to[a,b]$ such that f is one-one

(d) There exists a continuous function $f:(a,b)\to[a,b]$ such that f is onto

35. Let $a,b,c\in\mathbb{R}$ such that a<b<c. Which of the following is/are true for any continuous function $f:\mathbb{R}\to\mathbb{R}$ satisfying $f(a)=b, f(b)=c$ and $f(c)=a$?

(a) There exist $\alpha\in(a,c)$ such that $f(\alpha)=\alpha$

(b) There exist $\beta\in(a,b)$ such that $f(\beta)=\beta$

(c) There exists $\gamma\in(a,b)$ such that $(f\circ f)(\gamma)=\gamma$

(d) There exists $\delta\in(a,c)$ such that $(f\circ f\circ f)(\delta)=\delta$

36. Let V be a non-zero vector space over a field F. Let $S\subset V$ be a non-empty set. Consider the following properties of S:

(I) For any vector space W over F, any map $f:S\to W$ extends to a linear map from V to W.

(II) For any vector space W over F and any two lienar maps $f,g:V\to W$ satisfying $f(s)=g(s)$ for all $s\in S$ we have $f(v)=g(v)$ for all $v\in V$,

(III) S is linearly independent

(IV) The span of S is V

Which of the following statement (s) is/are True ?

(a) (I) implies (IV) (b) (I) implies (III) (c) (II) implies (III) (d) (II) implies (IV)

37. If $s_n = \frac{(-1)^n}{2^n+3}$ and $t_n = \frac{(-1)^n}{4n-1}, n = 0,1,2,...,$ then

(a) $\sum_{n=0}^{\infty} s_n$ is absolutely convergent

(b) $\sum_{n=0}^{\infty} t_n$ is absolutely convergent

(c) $\sum_{n=0}^{\infty} s_n$ is conditionally convergent

(d) $\sum_{n=0}^{\infty} t_n$ is conditionally convergent

38. Let f be a real valued function of a real variable, such that $\left|f^{(n)}(0)\right| \le K$ for all $n \in \mathbb{N}$, where K> 0. Which of the following is/are true ?

(a) $\left|\frac{f^{(n)}(0)}{n!}\right|^{\frac{1}{n}} \to 0$ as $n \to \infty$

(b) $\left|\frac{f^{(n)}(0)}{n!}\right|^{\frac{1}{n}} \to \infty$ as $n \to \infty$

(c) $f^{(n)}(x)$ exists for all $x \in \mathbb{R}$ and for all $n \in \mathbb{N}$

(d) The series $\sum_{n=1}^{\infty} \frac{f^{(n)}(0)}{(n-1)!}$ is absolutely convergent

39. Let G be a group with identity e. Let H be an abelian non-trivial proper subgroup of G with the property that $H \cap gHg^{-1} = \{e\}$ for all $g \notin H$. $K = \{g \in G; gh = hg \forall h \in H\}$, then

(a) K is a proper subgroup of H
(b) H is a proper subgroup of K
(c) K = H
(d) There exists no abelian subgroup $L \subseteq G$ such that K is a proper subgroup of L

40. Let S be that part of the surface of the paraboloid $z = 16 - x^2 - y^2$ which is above the plane z = 0 and D be its projection on the xy- plane. Then the area of S equals

(a) $\iint_D \sqrt{1+4(x^2+y^2)}\,dxdy$

(b) $\iint_D \sqrt{1+2(x^2+y^2)}\,dxdy$

(c) $\int_0^{2\pi}\int_0^4 \sqrt{1+4r^2}\,drd\theta$

(d) $\int_0^{2\pi}\int_0^4 \sqrt{1+4r^2}\,rdrd\theta$

SECTION-C

[Numerical Answer Type (NAT)]

Q.41 – Q.50 carry ONE mark each.

41. Let $\phi : S_3 \to S^1$ be a non-trivial non- injective group homomorphism. Then the number of elements in the kernel of ϕ is___________

42. Let $f : \mathbb{R} \to \mathbb{R}$ be such that f, f', f'' are continuous functions with $f > 0,\ f' > 0$ and $f'' > 0$. Then $\lim_{x\to\infty} \frac{f(x)+f'(x)}{2}$ is ___________

43. Let $S = \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$ and $f : S \to \mathbb{R}$ be defined by $f(x) = \frac{1}{x}$, then

$\max\left\{ \delta : \left| x - \frac{1}{3} \right| < \delta \Rightarrow \left| f(x) - f\left(\frac{1}{3}\right) \right| < 1 \right\}$ is ___________(upto two decimal places)

44. Let $f(x, y) = 0$ be a solution of the homogeneous differential equation $(2x+5y)\,dx - (x+3y)\,dy = 0$

If $f(x+\alpha, y-3) = 0$ is a solution of the differential equation $(2x+5y-1)\,dx + (2-x-3y)\,dy = 0$ then the value of α is ___________

45. Consider the real vector space $P_{2020} = \left\{ \sum_{i=0}^{n} a_i x^i ; a_i \in \mathbb{R} \text{ and } 0 \le \text{n} \le 2020 \right\}$. Let W be the subspace given by

$W = \left\{ \sum_{i=0}^{n} a_i x^i \in P_{2020} ; a_i = 0 \text{ for all odd } i \right\}$, Then the dimension of W is ___________

46. Let $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ and S be the sphere given by $(x-2)^2 + (y-2)^2 + (z-2)^2 = 4$. If $\hat{n}$ is the unit outward normal to S, then $\frac{1}{\pi}\iint_S \vec{F}\cdot\hat{n}\,dS$ is ___________

47. If $\int_0^1 \int_{2y}^2 e^{x^2}\,dx\,dy = k\left(e^4 - 1\right)$, then k equals___________

48. Let $x_n = n^{\frac{1}{n}}$ and $y_n = e^{1-x_n}, n \in \mathbb{N}$. Then the value of $\lim_{n\to\infty} y_n$ is ___________

49. Consider the differential equation $\frac{dy}{dx} + 10y = f(x), x > 0$. Where $f(x)$ is a continuous function such that $\lim_{x\to\infty} f(x) = 1$. Then the value of $\lim_{x\to\infty} y(x)$ is ___________

50. Let $f(x, y) = e^x \sin y, x = t^3 + 1$ and $y = t^4 + t$. Then $\frac{df}{dt}$ at $t = 0$ is__________ (upto two decimal places)

Q.51 – Q.60 carry TWO marks each.

51. Let $T:\mathbb{R}^7 \to \mathbb{R}^7$ be a linear tranformation with nullity (T) = 2. Then, the minimum possible value for Rank (T^2) is ___________

52. Let $M=\begin{bmatrix} 9 & 2 & 7 & 1 \\ 0 & 7 & 2 & 1 \\ 0 & 0 & 11 & 6 \\ 0 & 0 & -5 & 0 \end{bmatrix}$. Then, the value of $\det((8I-M)^3)$ is ___________

53. Consider the expansion of the function $f(x)=\dfrac{3}{(1-x)(1+2x)}$ in powers of x, that is valid in $|x|<\dfrac{1}{2}$. Then the coefficient of x^4 is___________

54. Suppose that G is a group of order 57 which is NOT cyclic. If G contains a unique subgroup H of order 19, then for any $g \notin H, o(g)$ is ___________

55. The minimum value of the function $f(x,y)=x^2+xy+y^2-3x-6y+11$ is___________

56. Let C be the boundary of the square with vertices (0, 0), (1, 0), (1, 1) and (0, 1) oriented in the counter clockwise sense. Then the value of the line integral $\oint_C x^2y^2dx+(x^2-y^2)dy$ is ___________ (upto two decimal places)

57. Let $f(x)=\sqrt{x}+\alpha x, x>0$ and $g(x)=a_0+a_1(x-1)+a_2(x-1)^2$ be the sum of first three terms of the Taylor series of $f(x)$ around $x=1$. If $g(3)=3$, then α is___________

58. If $x^2+xy^2=c$ where $c\in\mathbb{R}$, is the general solution of the exact differential equation $M(x,y)dx+2xydy=0$ then M(1, 1) is ___________

59. Let $f:\mathbb{R}\to\mathbb{R}$ be a differentiable function with $f'(x)=f(x)$ for all x, Suppose that $f(\alpha x)$ and $f(\beta x)$ are two non-zero solution of the differential equation $4\dfrac{d^2y}{dx^2}-p\dfrac{dy}{dx}+3y=0$ satisfying $f(\alpha x)f(\beta x)=f(2x)$ and $f(\alpha x)f(-\beta x)=f(x)$ then the value of p is ___________

60. The sum of the series $\dfrac{1}{2(2^2-1)}+\dfrac{1}{3(3^2-1)}+\dfrac{1}{4(4^2-1)}+....$ is ___________

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