1

Sets, Relation, Function

Defintion : (Set) : A set is a well defined collection of distinct objects. These objects are called elements of the set. A set is denoted by capital letters.

Example : The collection of rivers of India. The collection of natural numbers. The collection of vowels in the english alphabet. The collection of positive divisior of 42.

There are two forms of sets

(i) Roster form (Tabular form)

(ii) Set-builder form

Roster Form (Tabular Form) : In this form, we write every element of a set in a curley braces seperated by commas. Elements are not repeated and order of the elements does not matter.

For example, The collection of letters of the word "MATHEMATICS" = {M, A, T, H, E, I, C, S}

Set-builder form : In this form, all the elements of the set are denoted by a single letter (like x, y, z etc) followed by semi colon and then we write the characteristic property of the elements of the set.

For example, The collection of even integers = $\{x : x$ is an even integer$\}$

Example 1: Write the following sets in the roster form.

(i) A = $\{x|x$ is a positive integer less then 10 and $2^x - 1$ is an odd number$\}$

(ii) B = $\{x|x$ is real number and $x^2 + 7x - 8 = 0\}$

Solution :

(i) $2^1 - 1 = 1$, $2^2 - 1 = 3$, $2^3 - 1 = 7$

$2^4 - 1 = 15$, $2^5 - 1 = 31$, $2^6 - 1 = 63$

$2^7 - 1 = 127$, $2^8 - 1 = 255$, $2^9 - 1 = 511$

Thus, $2^x - 1$ is always an odd number for all positive integer value less than 10.

A = { 1, 2, 3, 4, 5, 6, 7, 8, 9}

(ii) $x^2 + 7x - 8 = 0$

$\Rightarrow\ x^2 + 8x - x - 8 = 0$

$\Rightarrow\ x(x+8) - 1(x+8) = 0$

$\Rightarrow\ (x-1)(x+8) = 0$

$\Rightarrow\ x = 1, -8$

$B = \{+1, -8\}$

TYPE OF SETS

(i) The empty set : A set containing no elements is called an empty set. It is denoted by ϕ or { }.

(ii) Finite set : A set containing finite number of elements is called finite set.

(iii) Infinite set : A set which is not finite is called infinite set

(iv) Singleton set : A set containing only one elements is called a singleton set.

Ex. The collection of even prime number is/are

(a) Empty set (b) Finite set (c) Infinite set (d) Singleton set

Soln. Let A = set of even prime number, A = {2}

A is a singleton set. Hence finite set.

Correct options are (b) and (d)

SUBSETS:

A set X is subset of set Y if every elements of set X is also an element of set Y.

We write $X \subseteq Y$ if $x \in X \Rightarrow x \in Y$

Number system:

$\mathbb{N}$ = set of Natural Number = {1, 2, 3, 4, ...}

$\mathbb{Z}$ = set of Integers = $\{0, \pm 1, \pm 2, \pm 3, ...\}$

$\mathbb{Q}$ = set of Rational number = $\left\{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\right\}$

= $\{x : x$ has terminating or non terminating repeating decimal expansion$\}$

$\mathbb{T} = \mathbb{Q}'$ = set of irrational number

= $\{x : x$ is not a rational number$\}$

= $\{x : x$ has non terminating non repeating decimal expansion$\}$

We observe that,

$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$ and $\mathbb{Q}' \subseteq \mathbb{R}$

EQUAL SETS :

Two or more sets are said to be equal if they have exactly same element.

$A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$

Ex. Let A = Collection of letters of the word “MATHEMATICS”

B = Collection of letters of the word “MATHS”

Then

(a) A = B (b) $A \subseteq B$ (c) $B \subseteq A$ (d) $A \neq B$

Soln. $A = \{M, A, T, H, E, I, C, S\}$

$B = \{M, A, T, H, S\}$

Every element of set B is in set A.

$B \subseteq A$ and $A \neq B$

Correct options are (c) and (d)

Intervals : Let $a,b \in \mathbb{R}$ and $a < b$. Then

$(a,b) = \{x \in \mathbb{R} : a < x < b\}$

$[a,b) = \{x \in \mathbb{R} : a \le x < b\}$

$(a,b] = \{x \in \mathbb{R} : a < x \le b\}$

$[a,b] = \{x \in \mathbb{R} : a \le x \le b\}$

POWER SET :

The collection of all subsets of a set A is called the power set of A. It is denoted by P(A).If A has n elements then P(A) has 2^n elements.

Ex. If $A = \{\phi,1\}$, then $P(A)$ is

(a) $\{\phi,1\}$ (b) $\{\phi,\{1\}\}$ (c) $\{\phi,\{1\},\{\phi\}\}$ (d) $\{\phi,\{\phi\},\{1\},\{\phi,1\}\}$

Soln. A has 2 elements then P(A) has 4 elements, A has 4 subsets that are $\phi,\{\phi\},\{1\},\{\phi,1\}$

$P(A) = \{\phi,\{\phi\},\{1\},\{\phi,1\}\}$

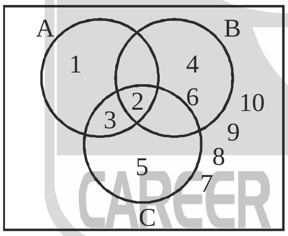
Correct option is (d)

UNIVERSAL SET :

For some given sets, A set which contains all the elements of given sets is called universal set. It is denoted by U.

Venn diagrams : Venn diagrams are the diagrams which represents the relationship between sets. It is visual representation of sets in which rectangle represents universal sets and circle represents subsets of universal sets.

Exmaple : $A = \{1,2,3\}, B = \{2,4,6\}, C = \{2,3,5\}, U = \{1,2,3,4,5,6,7,8,9,10\}$



Operation on sets

(1) Union of sets :

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

That is, $x \in A \cup B \Leftrightarrow x \in A$ or $x \in B$

Properties of Union :

(i) $A \cup B = B \cup A$

(ii) $(A \cup B) \cup C = A \cup (B \cup C)$

(iii) $A \cup \phi = \phi \cup A = A$

(iv) $A \cup A = A$

(v) $U \cup A = A \cup U = U$

(iv) $A \subseteq B$ then $A \cup B = A$

(2) Intersection of sets: Let A and B are two sets, then intersection of sets is defined as

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

i.e., $x \in A \cap B \Leftrightarrow x \in A$ and $x \in B$

Properties of intersection:

(i) $A \cap B = B \cap A$

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

(iii) $\phi \cap A = A \cap \phi = \phi$

(iv) $U \cap A = A \cap U = A$ (where U is universal set)

(v) $A \cap A = A$

(vi) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(vii) $A \cup (B \cap C) = (A \cup B) \cap (B \cup C)$

(viii) $A \subseteq B$ then $A \bigcap B = A$

Disjoint (Mutually Exclusive) sets : Two sets are disjoint if $A \cap B = \phi$

(3) Difference of sets : Let A and B are two sets, then difference of sets is defined as

$A \cap B^C = A - B = \{x : x \in A \text{and } x \notin B\}$ and $B \cap A^C = B - A = \{x : x \in B \text{and } x \notin A\}$.

(4) Complement of a set : Let U be universal set and A is subset of U. Then

$A' = A^C = U - A = \{x \in U : x \notin A\}$

Properties of complement :

(i) $A \cup A' = U$

(ii) $A \cap A' = \phi$

(iii) $(A \cup B)' = A' \cap B'$

(iv) $(A \cap B)' = A' \cup B'$

(v) $(A')' = A$

(vi) $U' = \phi$

(vii) $\phi' = U$

Application of operations : If A, B, C are finite sets. Then

(a) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(b) $n(A - B) = n(A) - n(A \cap B)$

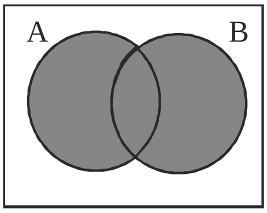
(c) $n(B - A) = n(B) - n(A \cap B)$

(d) $n(A') = n(U) - n(A)$

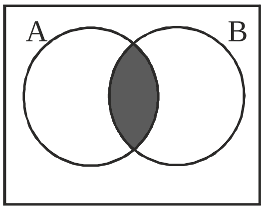
(e) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

(f) If $A \cap B = \phi$ then $n(A \cup B) = n(A) + n(B)$

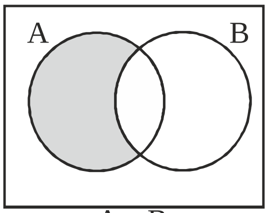
Venn-diagrams : If A, B, C are finite sets then



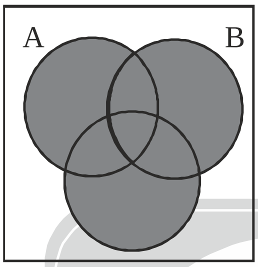
$A \cup B$



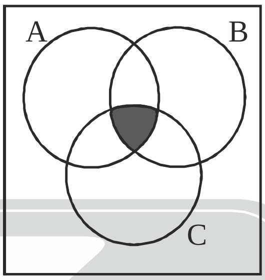
$A \cap B$



$A - B$



$A \cup B \cup C$



$A \cap B \cap C$

Ex. The operation $(P \cap Q) \cup (P \cap R)$ for sets P, Q and R can also be written as

(a) $P \cup (Q \cup R)$ (b) $P \cap (Q \cup R)$ (c) $P \cap (Q \cap R)$ (d) $P \cup (Q \cap R)$

Soln. $(P \cap Q) \cup (P \cap R) = P \cap (Q \cup R)$

Correct option is (b)

Ex. A hospital has 35 patients, 24 of which are HIV + and 16 have TB infection. All patients have at least one of the two infections. The number of patients with both HIV and TB infections is

(a) 5 (b) 8 (c) 9 (d) 11

Soln. A : HIV +

B : TB

$n(A \cup B) = 35, \quad n(A) = 24, \quad n(B) = 16$

$n(A \cap B) = n(A) + n(B) - n(A \cup B) = 24 + 16 - 35 = 40 - 35 = 5$

Correct option is (a)

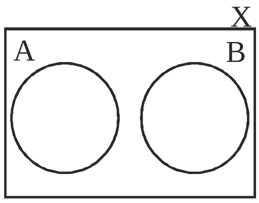
Ex. Let A, B be two subsets of a set X such that $A \cap B = \phi$. Then

(a) $A \subseteq X - B$ (b) $(X - A) \cap (X - B) = \phi$

(c) $(X - A) \cup (X - B) = \phi$ (d) $(X - B) \subseteq A$

Soln. $A \subseteq X, B \subseteq X, A \cap B = \phi$

$\Rightarrow\ A \subseteq X - B$ but $X - B \not\subseteq A$



Also, $(X - A) \cap (X - B) \neq \phi$

Also, $(X - A) \cup (X - B) = X \neq \phi$

Correct option is (a)

Ex. In a survey, 96 people liked straw berries,98 liked raspberries and 120 liked blue berries, 18 people liked only strawberries, 20 liked only respberries and 24 liked only blueberries, 38 people liked all the three. How many people liked raspberries and blueberries but not strawberries?

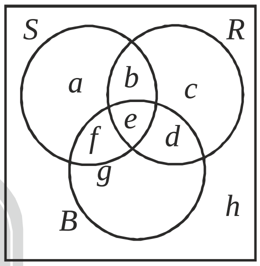
(a) 29 (b) 30 (c) 31 (d) 32

Soln. $a = 18, c = 20, g = 24, e = 38$

$a + b + e + f = 96$ .....(i)

$b + c + d + e = 98$ .....(ii)

$d + e + f + g = 120$ .....(iii)



equation (iii) $\Rightarrow d + f = 120 - 62 = 58$ .....(iv)

equation (ii) $\Rightarrow b + d = 98 - 58 = 40$ .....(v)

equation (i) $\Rightarrow b + f = 96 - 56 = 40$ .....(vi)

equation (iv) + (v) + (vi)

$2(d + f + b) = 138$

$\Rightarrow d + f + b = 69$ .....(vii)

$\Rightarrow d + 40 = 69$

$\Rightarrow d = 29$

Correct answer is (29)

Ex. Let $\mathbb{R}$ be the set of real numbers. Consider the set $P = \{x \in \mathbb{R} : (x-1)(x^2+1) = 0\}, Q = \{x \in \mathbb{R} : x^2 - 9x + 2 = 0\}$ and $S = \{x \in \mathbb{R} : x = 5y \text{ for some } y \in \mathbb{R}\}$. Then $(P \cap S) \cup Q$ contains

(a) Exactly two elements (b) Exactly three elements

(c) Exactly four elements (d) infinitely many elements

Soln. $(x-1)(x^2+1) = 0 \quad \Rightarrow x = 1$ (since $x \in \mathbb{R}$)

$$x^2 - 9x + 2 = 0 \Rightarrow x = \frac{9 \pm \sqrt{81-8}}{2} = \frac{9 \pm \sqrt{73}}{2}$$

$$P = \{1\}, Q = \left\{\frac{9+\sqrt{73}}{2}, \frac{9-\sqrt{73}}{2}\right\}, S = \mathbb{R}$$

$P \cap S = \{1\}$

$$(P \cap S) \cup Q = \left\{1, \frac{9+\sqrt{73}}{2}, \frac{9-\sqrt{73}}{2}\right\}$$

$(P \cap S) \cup Q$ has three elements

Correct option is (b)

Ex. If P = {1, 2, –1, 3}, Q = {0, 4, 1, 3} and R = {1, 6, 7} then $P \cap (Q \cup R)$ is

(a) {1, 2} (b) {1, 3} (c) {2, 1} (d) {2, 3}

Soln. $Q \cup R = \{0,1,3,4,7,6\}$

$P \cap (Q \cup R) = \{1,3\}$

Correct option is (b)

Ex. In a group of 50 persons, 14 drink tea but not coffee and 30 drink tea.How many persons drink tea and coffee both

(a) 30 (b) 36 (c) 16 (d) 20

Soln. $n(T - C) = 14, n(T) = 30$

$\Rightarrow n(T) - n(T \cap C) = 14$

$\Rightarrow 30 - n(T \cap C) = 14$

$\Rightarrow n(T \cap C) = 16$

Correct option is (c)

Ex. In a class of 50 students, 10 study Biology, 30 Chemistry and 25 physics. 11 students study both physics and chemistry, 9 students study Biology and Chemistry and 7 students study Biology and physics. The number of students who study all the three subjects is

(a) 1 (b) 2 (c) 3 (d) 4

Soln. $n(B) = 11$, $n(C) = 30$, $n(P) = 25$,

$n(B \cup C \cup P) = 50$

$n(P \cap C) = 11,\ n(B \cap C) = 9,\ n(B \cap P) = 7$

We have

$n(B \cup C \cup P) = n(B) + n(C) + n(P) - n(B \cap C) - n(C \cap P) - P(B \cap P) + P(B \cap C \cap P)$

$\Rightarrow 50 = 10 + 30 + 25 - 11 - 9 - 7 + n(B \cap C \cap P)$

$\Rightarrow n(B \cap C \cap P) = -73 + 27 + 50 = 77 - 73 = 4$

Correct option is (d)

Ex. In a survey of 25 owners of dogs and cat, 20 owned dogs and 10 owned cats. The number of people who owned both dogs and cats was _______.

Soln. $n(C \cup D) = 25$

$n(D) = 20, n(C) = 10$

$n(C \cap D) = n(D) + n(C) - n(C \cup D) = 20 + 10 - 25 = 5$

Correct answer is (5)

Cartesion Product of sets: Let A and B are two sets, then cartesion Product of sets is defined as

$A \times B = \{(x, y) : x \in A \text{ and } y \in B\}$ & $B \times A = \{(y, x) : y \in B \text{ and } x \in A\}$.

Also, $A \times A = \{(x, y) : x, y \in A\}$ & $B \times B = \{(x, y) : x, y \in B\}$.

Similarly, $A \times B \times C = \{(a,b,c) : a \in A, b \in B, c \in C\}$.

Remarks: (i) $(x, y) = (u, v) \Leftrightarrow x = u$ and $y = v$

(ii) $n(A \times B) = n(A).n(B)$

(iii) $n(A \times B) = n(B \times A)$ but $A \times B \neq B \times A$

RELATION :

Any subset of the cartesion product $A \times B$ is called a relation form a non-empty set A to a non-empty set B.

i.e., $R : A \to B$ is a relation if $R \subseteq A \times B$.

(a) The set of all first-elements and second element are called Domain and Range of relation R respectively.

(b) Set B is called Co-domain of relation R and Range $\subseteq$ Co-Domain

(c) If $n(A) = p$ and $n(B) = q$ then $n(A \times B) = pq$ and number of relation from set A to set B is 2^{pq}.

Remark: A relation can be represented by roster form or set-builder form or by arrow diagram.

Ex. Let A = {1, 2, 3, 4, 5}, B = {3, 4, 5, 6, 7,8, 9, 10} and R be relation from set A to set B defined by $R = \{(x, y) : y = 2x - 3\}$. Write R in roster form. Write domain, Co-domain and Range of relation. Visualise R on arrow diagram.

Soln. Since $x \in A$ and $y \in B$

$y = 2x - 3$

$R = \{(3, 3), (4, 5), (5, 7)\}$ is roster form

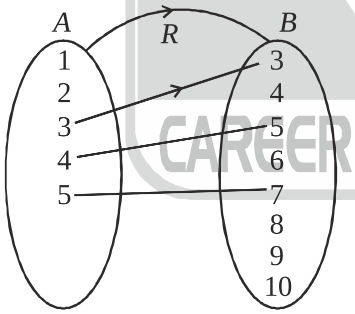
Domain = set of all first element

$= \{3, 4, 5\} \subseteq A$

Range = set of all second element

$= \{3, 5, 7\} \subseteq B$

Co-Domain = B = {3, 4, 5, ..., 10}



Relation on a set : A relation on set A is a subset of A × A.

TYPES OF RELATIONS

(i) Empty relation : A relation on set A in which no element of set A is related to any element of set A.

i.e. $R = \phi \subseteq A \times A$

(ii) Universal relation : A relation on set A in which every element of set A is related to all elements of set A.

i.e. $R = A \times A \subseteq A \times A$

(iii) Identity relation : A relation on set A in which every element of set A is related to itself only and no element is related to other.

i.e. $R = \{(a, a) : a \in A\}$

(iv) Reflexive relation : A relation $R : A \to A$ is reflexive if $(a,a) \in R \forall a \in A$

(v) Symmetric relation : A relation $R : A \to A$ is symmetric if $(a,b) \in R$ then $(b,a) \in R$ for all $a,b \in A$.

(vi) Transitive relation: A relation $R : A \to A$ is transitive if $(a,b) \in R, (b,c) \in R$ then $(a,c) \in R \ \forall a,b,c \in A$

(vii) Equivalence relation : A relation $R : A \to A$ is equivalence relation if R is reflexive, symmetric and transitive relation.

Equivalence classes : An equivalence relation $R : A \to A$ partition the set A into disjoint subsets, called equivalence classes.

Collection of equivalence class is called a partition of the set. Union of all equivalence classes gives the whole set A.

Ex. The total number of relations on set $A = \{a, b, c\}$ is equal to

(a) 2^3 (b) 2^6 (c) 2^9 (d) 2^{12}

Soln. $n(A) = 3 \Rightarrow n(A \times A) = 9$

Total number of subsets of $A \times A = 2^9$

Total number of relation on set $A = 2^9$

Correct opiton is (c)

Ex. In the set of real numbers, the relation 'greater than' is

(a) reflexive (b) symmetric (c) transitive (d) None

Soln. Let R be a relation on $\mathbb{R}$.

$R : \mathbb{R} \to \mathbb{R}$

$\text{R} = \{(a,b) : a \text{ is 'greater than' } b\} = \{(a,b) : a > b\}$

Since $(a,a) \notin R$ because $a > a$ is false for all $a \in \mathbb{R}$

If $a > b$ then $b \not> a$.

Therefore, R is not symmetric.

If $(a,b) \in R, (b,c) \in R$

$\Rightarrow \ a > b, b > c$

$\Rightarrow \ a > c$

$\Rightarrow \ (a,c) \in R$ for all $a,b,c \in R$

R is transitive relation

Correct option is (c)

Ex. A relation R is defined on the set $\mathbb{Z}$ of integers by aRb if and only if $(a + b)$ is an even integer. Then R is

(a) reflexive but not transitive (b) Symmetric but not reflexive

(c) transitive but not symmetric (d) Equivalence

Soln. $R : \mathbb{Z} \to \mathbb{Z}$

$R = \{(a,b) : a + b \text{ is an even integer}\}$

Let $a \in \mathbb{Z} \Rightarrow a + a = 2a$ is even integer

$\Rightarrow (a,a) \in R \quad \forall a \in \mathbb{Z}$

R is reflexive relation.

Let $a, b \in \mathbb{Z}$ and $(a,b) \in R \Rightarrow a + b$ is even integer

$\Rightarrow b + a$ is even integer

$\Rightarrow (b,a) \in R \ \forall a, b \in \mathbb{Z}$

R is symmetric relation

Let $a, b, c \in \mathbb{Z}$ and $(a,b) \in R, (b,c) \in R$

$\Rightarrow a + b$ is even integer and $b + c$ is even integer.

(odd + odd = even or even + even = even)

$\Rightarrow a + c$ is even integer

$\Rightarrow (a,c) \in R \ \ \forall a, c \in \mathbb{Z} \Rightarrow R$ is transitive relation

$\therefore$ R is equivalence relation

Correct option is (d)

Ex. Let X and Y be two sets such that X and Y have 10 elements are in common. Then the number of elements common to $X \times Y$ and $Y \times X$ is

(a) 20 (b) 50 (c) 100 (d) 200

Soln. $n(X \cap Y) = 10$

$n\{(X \times Y) \cap (Y \times X)\} = 10 \times 10 = 100$

Correct option is (c)

Ex. Let $\mathbb{N}$ be the set of all natural numbers. Consider the relation R on $\mathbb{N}$ given by $R = \{(m,n) : m - n$ is divisible by $2\}$. Then

(a) R is symmetric and transitive (b) R is symmetric but not transitive

(c) R is reflexive and not symmetric (d) R is reflexive and transitive

Soln. $R : \mathbb{N} \to \mathbb{N}$

$R = \{(m,n) : m - n$ is divisible by $2\}$

Let $m \in \mathbb{N} \Rightarrow m - m = 0$ is divisible by 2.

$\Rightarrow (m,m) \in R \ \forall m \in \mathbb{N}$

$\Rightarrow$ R is reflexive relation.

Let $m, n \in \mathbb{N}$ and $(m,n) \in R$

$\Rightarrow m - n$ is divisible by 2

$\Rightarrow n - m$ is also divisible by 2

$\Rightarrow (n,m) \in \ \forall m, n \in \mathbb{N}$

$\Rightarrow R$ is symmetric relation

Let $m, n, r \in \mathbb{N}$ and $(m,n) \in R, (n,r) \in R$

$\Rightarrow m - n$ is divisible by 2 and $n - r$ is divisible by 2

$\Rightarrow$ $m-r$ is divisible by 2

$\Rightarrow$ $(m,r)\in R\ \forall m,n,r\in\mathbb{N}$

$\therefore$ R is transitive relation

R is equivalence relation

Correct options are (a) and (d)

FUNCTIONS

Function : A relation from a set A to a set B is said to be a function if every element of set A has a unique image in set B. Every function is a relation but every relation is not a function.

If $n(A)=p$, $n(B)=q$ then Total number of relation from A to $B=2^{pq}$ and Total number of function from A to $B=q^{p}$

Set A is called domain of f and set B is called co-domain of f. The set $f(A)=\{f(x): x\in A\}$ is called range of function f and $f(A)\subseteq B$.

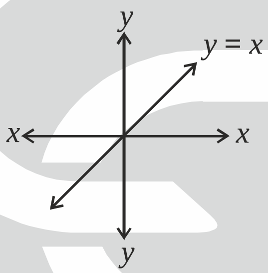
***Real function and Real valued function* :**

A function which has either $\mathbb{R}$ or a subset of $\mathbb{R}$ as its domain is called real function.

or A function which has either $\mathbb{R}$ or a subset of $\mathbb{R}$ as its range is called a real valued function.

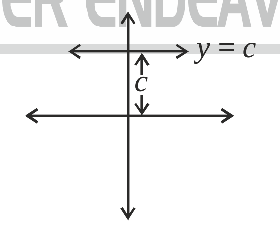
Some specific function

(i) Identity function : The function $f:\mathbb{R}\to\mathbb{R}$ defined by $y=f(x)=x$ is called the identity function Domain $=\mathbb{R}$, Range $=\mathbb{R}$.



(ii) Constant function : The function $f:\mathbb{R}\to\mathbb{R}$ defined by $y=f(x)=c$ where c is real constant, is called a constant function.

Domain $=\mathbb{R}$ Range $=\{c\}$



(iii) Polynomial function: A real valued function $f:\mathbb{R}\to\mathbb{R}$ defined by

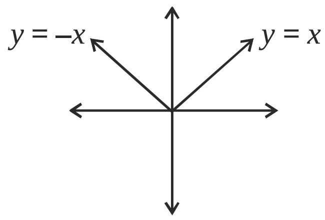
$y=f(x)=a_0+a_1x+a_2x^2+.......+a_nx^n$ where $n\in\mathbb{N}$ and and $a_0,a_1,a_2,......,a_n\in\mathbb{R}$ for each $x\in\mathbb{R}$, is called polynomial functions.

(iv) Rational function : A real function of the type $\dfrac{f(x)}{g(x)}$ where $f(x)$ and $g(x)$ are polynomial function.

Domain $=\{x\in\mathbb{R}: g(x)\neq 0\}$

(v) The Modulus function : A function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=|x|=\begin{cases} x, x\ge 0 \\ -x, x<0 \end{cases}$ called the modulus function.

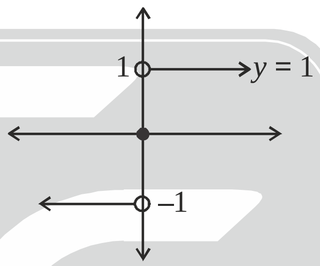
Domain = $\mathbb{R}$, Range = $[0,\infty)$



(vi) Signum function : The real function $f:\mathbb{R}\to\mathbb{R}$ defined by

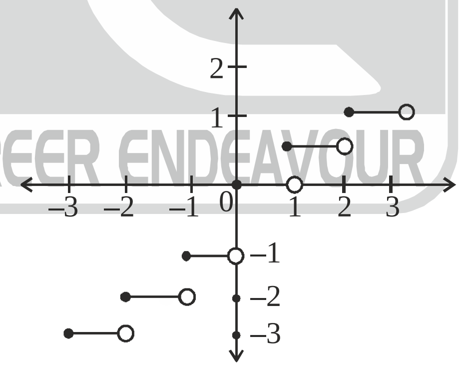
$$f(x)=\begin{cases} \frac{|x|}{x}, & x\ne 0 \\ 0, & x=0 \end{cases}=\begin{cases} 1 & \text{if } x>0 \\ 0 & \text{if } x=0 \\ -1 & \text{if } x<0 \end{cases}$$

is called the signum function. Domain = $\mathbb{R}$, Range = $\{1, 0, -1\}$



(vii) Greatest Integer function : The function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=[x]$ = greatest integer less than or equal to x, is called the greatest integer function.

Domain = $\mathbb{R}$, Range = $\mathbb{Z}$



(viii) Algebra of real function : Let $f:\mathbb{R}\to\mathbb{R},\ \ g:\mathbb{R}\to\mathbb{R}$ then

$(f+g)(x)=f(x)+g(x)$

$(f-g)(x)=f(x)-g(x)$

$(f.g)(x)=f(x).g(x)$

$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$

$(\alpha f)(x)=\alpha.f(x)$ where α is constant.

TYPES OF FUNCTION

(i) One - one function (Injective function) : A function $f : A \to B$ is said to be one-one if distinct elemensts of set A has distinct images in set B. i.e. If $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)\ \forall x_1, x_2 \in A$

i.e. If $f(x_1) = f(x_2) \Rightarrow x_1 = x_2\ \forall x_1, x_2 \in A$

(ii) Onto (Surjective) function) : A function $f : A \to B$ is said to be onto if $Range(f) = Co-Domain(f)$

i.e. $f(A) = B$

i.e. for every $y \in B$, there exists an $x \in A$ such that $f(x) = y$.

(iii) Bijective function : A function which is both one-one and onto function is called a bijective function.

(iv) Many one-function : A function which is not one-one function is called many one function.

(v) Into function : A function which is not onto function is called Into function

Ex. If $f(x) = \begin{cases} 3 & \text{when } -3 \le x \le -1 \\ -6x-3 & \text{when } -1 \le x \le 0 \\ 3x-3 & \text{when } 0 \le x \le 1 \end{cases}$

then the values of x for which $2f(x)+3=0$ are

(a) $\frac{1}{4}, \frac{1}{2}$ (b) $\frac{-1}{4}, \frac{1}{2}$ (c) $\frac{1}{4}, \frac{-1}{2}$ (d) $\frac{-1}{4}, \frac{-1}{2}$

Soln. $2f(x)+3=0$

$\Rightarrow f(x) = \frac{-3}{2}$

when $-1 \le x \le 0$ then $-6x-3 = \frac{-3}{2}$

$\Rightarrow -6x = \frac{3}{2}$

$\Rightarrow x = -\frac{1}{4}$

when $0 \le x \le 1$ then $3x-3 = \frac{-3}{2} \Rightarrow x = \frac{1}{2}$

Correct option is (b)

Ex. Let $1 < x < \infty$ and $f(x) = \log\left(\frac{x+1}{x-1}\right)$. Then $f\left(\frac{x^3+3x}{1+3x^2}\right)$ equals

(a) $f(x+3)$ (b) $f(x^2+3)$ (c) $2f(x)$ (d) $3f(x)$

Soln. $f\left(\frac{x^3+3x}{1+3x^2}\right) = \log\left(\frac{\frac{x^3+3x}{1+3x^2}+1}{\frac{x^3+3x}{1+3x^2}-1}\right) = \log\left(\frac{x^3+3x+1+3x^2}{x^3+3x-1-3x^2}\right) = \log\left(\frac{(x+1)^3}{(x-1)^3}\right) = 3\log\left(\frac{x+1}{x-1}\right) = 3f(x)$

Correct option is (d)

Ex. The number of functions from the set $\{a, b, c, d\}$ to the set $\{1, 2, 3\}$ is

(a) 12 (b) 36 (c) 64 (d) 81

Soln. Let A = $\{a, b, c, d\}$, B = $\{1, 2, 3\}$

$n(A) = 4, n(B) = 3$

The number of function from A to B = $(n(B))^{n(A)} = 3^4 = 81$

Correct option is (d)

Ex. Let $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=[x]$ and $g(x)=\frac{3-2x}{4}$, then

(a) f is one-one and onto (b) g is one-one but f is not one-one

(a) f is one-one and g is onto (d) Neither f is onto nor g is one-one

Soln. $f(1)=[1]=1$

$f(1.5)=[1.5]=1$

Since $1\neq 1.5$ but $f(1)=f(1.5)$

f is not one-one function

Range (f) = $\mathbb{Z}$ and Co-domain of $f=\mathbb{R}$

$\Rightarrow$ Range $\neq$ Co-Domain

$\therefore$ f is not onto

$g(x)$ is linear polynomial

$\Rightarrow g(x)$ is both one-one and onto

Correct option is (b)

Ex. The function $f(x)=\sin(x)$ is

(a) one to one and onto from $\left[0,\frac{\pi}{2}\right]$ *to* $[0,1]$. (b) one to one and onto from $[0,\pi]$ *to* $[0,1]$

(c) one to one and onto from $\left[0,\frac{\pi}{2}\right]$ *to* $\left[0,\frac{\pi}{2}\right]$ (d) one to one and onto form $[0,\pi]$ *to* $[0,\pi]$

Soln. (a) $f:\left[0,\frac{\pi}{2}\right]\to[0,1]$ and $f:(x)=\sin(x)$ Distinct elements of $\left[0,\frac{\pi}{2}\right]$ has distinct images and Range = Co-domain = [0, 1] Therefore f is both one-one and onto form $\left[0,\frac{\pi}{2}\right]$ *to* $[0,1]$

(b) $f:[0,\pi]\to[0,1]$ and $f(x)=\sin(x)$

$f(0)=f(\pi)=0$ but $0\neq\pi \Rightarrow f$ is not one-one

Range = Co-Domain = [0, 1] $\Rightarrow f$ is onto.

(c) $f:\left[0,\frac{\pi}{2}\right]\to\left[0,\frac{\pi}{2}\right]$

If $x_1\neq x_2 \Rightarrow f(x_1)\neq f(x_2)\ \forall x_1,x_2\in\left[0,\frac{\pi}{2}\right]$

$\therefore$ f is one-one function.

Range = [0, 1] $\neq$ Co-domain

$\therefore$ f is not onto.

(d) $f:[0,\pi]\to[0,\pi]$ and $f(x)=\sin(x)$

$f:(0)=f(\pi)=0$ but $0\neq\pi\Rightarrow f$ is not one-one.

Range $=[0,1]\neq$ Co-domain.

f is not onto.

Correct option is (a)

Ex. Let $f:\{1,2,3,4,5\}\to\{a,b,c,d\}$ be a function. Which of the following statement is true?

(a) f can be one-one function (b) f is onto function

(c) f cannot be one-one function (d) f cannot be onto function

Soln. $f:A\to B$ where $A=\{1,2,3,4,5\}$, $B=\{a,b,c,d\}$ and finite set and $n(A)>n(B)$

At least 2 elements in A has same image.

$\Rightarrow$ f cannot be one-one function

Correct option is (c)

Ex. The signum function $f:\mathbb{R}\to\mathbb{R}$ given by $f(x)=\begin{cases}1 & \text{if } x>0\\ 0 & \text{if } x=0\\ -1 & \text{if } x<0\end{cases}$ is

(a) one-one and onto (b) one-one but not onto

(c) onto but not one-one (d) Neither one-one nor onto

Soln. $f(1)=f(2)$ *but* $1\neq 2$

f is not one-one function

Range $=\{1,0,-1\}\neq$ Co – Domain

f is not onto

Correct option is (d)

Ex. The function $f:[-2,1]\to[0,4]$ defined by $f(x)=x^2$

(a) surjective but not injective (b) not surjective but injective

(c) surjective as well as injective (d) neither surjective nor injective

Soln. $f(-1)=f(1)$ but $-1\neq 1$

f is not one-one (injective)

Range $=f([-2,1])=[0,4]=$ Co-Domain, f is onto (surjective).

Thus, f is surjective but not injective.

Correct option is (a)

Ex. If $f(3)=15$ and $f(5)=45$, which of the following could be $f(x)$?

(a) $4x+3$ (b) $2x^2-2x$ (c) $2x^2-x$ (d) $2x^2-5$

Soln. $f(x)=2x^2-x$

$f(3)=2(3)^2-3=18-3=15$

$f(5)=2(5)^2-5=50-5=45$

Correct option is (c)

Ex. Which of the following describes the relationship between A and B as shown in the pairs of numbers in the table below.

A	2	3	4	5
B	5	10	17	26

(a) $B=A+4$ (b) $B=2A+1$ (c) $B=3A-1$ (d) $B=A^2+1$

Soln. $B = A^2 + 1$ satisfies the table completely.

Correct option is (d)

Ex. For the function $f : x \to x^2$ with domain $\{x : -3 \le x \le 3\}$, the range is

(a) $\{y : 0 \le x \le 9\}$ (b) The set of all real number

(c) $\{y : -9 \le y \le 9\}$ (d) $\{y : y \le 3\}$

Soln. $A = \{x : -3 \le x \le 3\}$

$f(x) = x^2$

$f(A) = \{y : 0 \le y \le 9\}$

Correct option is (a)

Ex. The two function $y = x^2 - 2$ and $y = 4x - 6$ intersect at

(a) (4, 10) and (4, 10) (b) (4, 10)

(c) (2, 2) and (4, 10) (d) (2, 2)

Soln. $y = x^2 - 2$ & $y = 4x - 6$

Then $x^2 - 2 = 4x - 6$

$\Rightarrow x^2 - 4x + 4 = 0$

$\Rightarrow (x-2)^2 = 0$

$\Rightarrow x = 2$

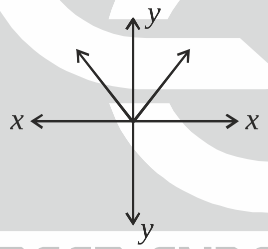
$\Rightarrow y = (2)^2 - 2$

$y = 2$

Intersection point is (2, 2)

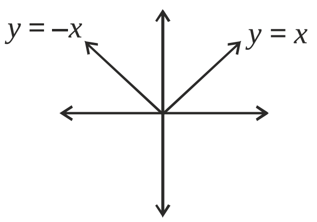
Correct option is (d)

Ex. Which one of the following is the correct expression for the function $f(x)$ shown in the figure below ?



(a) $f(x) = \dfrac{1}{x}$ (b) $f(x) = |x|$ (c) $f(x) = x^2$ (d) $f(x) = \dfrac{1}{x^2}$

Soln. $f(x) = |x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$



Correct option is (b)

Ex. What is the output range of the function $y = 1 - e^x$ for input values in the interval $-\infty < x < \infty$?

(a) $-\infty < y < \infty$ (b) $0 < y < \infty$ (c) $-\infty < y < 1$ (d) $0 < y < 1$

Soln. $e^x > 0 \ \forall x \in \mathbb{R}$

$\Rightarrow \quad -e^x < 0 \ \forall x \in \mathbb{R}$

$\Rightarrow \quad 1-e^x < 1 \ \forall x \in \mathbb{R}$

$\Rightarrow \quad y < 1 \ \forall x \in \mathbb{R} \Rightarrow -\infty < y < 1 \ \forall x \in \mathbb{R}$

$\Rightarrow \quad y \in (-\infty, 1)$

Correct option is (c)

Ex. The total number of mapping from the set {1, 2} to the set {3, 4, 5, 6, 7} is __________

Soln. $f : A \to B$

$A = \{1,2\} \qquad B = \{3,4,5,6,7\}$

$n(A) = 2, \qquad n(B) = 5$

Number of function = $5^2 = 25$

Correct answer is 25

Ex. If P = 10 and Q = 4 then the value of 1 + P – Q $(1 - e^{-x})$ when $x = \infty$ *is* ________.

Soln. $1 + P - Q(1-e^{-x})$

$= 1 + 10 - 4(1 - e^{-\infty})$

$= 11 - 4 \times (1 - 0)$

$= 11 - 4 = 7$

Correct answer is 7

Ex. The function $f(x) = x^3 - 1$ has the real roots

(a) –1, 0, 1 (b) –1, 1 (c) 0, 1 (d) 1

Soln.

$f(x) = 0$

$\Rightarrow \quad x^3 - 1 = 0$

$\Rightarrow \quad x^3 = 1$

$\Rightarrow \quad x = 1, \dfrac{1 \pm \sqrt{3}i}{2}$

$\Rightarrow \quad x = 1$ is only real root

Correct option is (d)

Composition of Function:

Let $f : A \to B$ and $g : B \to C$ be two function. Then the composition of *f* and *g* is denoted by *gof* and defined by $gof : A \to C$ *and* $gof(x) = g(f(x)) \ \forall x \in A$.

(a) If $f : A \to B$ and $g : B \to C$ are one-one onto.Then $gof : A \to C$ is also one-one or onto respectively.

(b) If $f : A \to B$ and $g : B \to C$ are function such that *gof* is one-one then *f* is one-one

(c) If $f : A \to B$ and $g : B \to C$ are function such that *gof* is onto then *g* is onto.

Ex. Let $\mathbb{Z}$ be the set of all integer and *f* and *g* are one-one mapping from $\mathbb{Z}$ *to* $\mathbb{Z}$.

If $\begin{cases} f(g(n)) = g(n+1)+1 & for\ even\ n \\ g(f(n)) = f(n-1)-1 & for\ odd\ n \end{cases}$ and $f(1) = 3$ then

(a) $g(2)=0$ (b) $f(3)=2$ (c) $g(2)=1$ (d) $f(3)=1$

Soln. Put $n=1 \Rightarrow g(f(1))=f(0)-1 \Rightarrow g(3)=f(0)-1$

Put $n=2 \Rightarrow f(g(2))=g(3)+1=f(0)-1+1=f(0)$

$\Rightarrow f(g(2))=f(0)$

$\Rightarrow g(2)=0$ because *f* is one-one.

Correct option is (a)

Ex. If $h = gof$,the composite of two function f and g where $f:\mathbb{R}\to\mathbb{R}$, $f(x)=x+2x^3$, and $g:\mathbb{R}\to\mathbb{R}$, $g(x)=1+x^2$ then $h(x)$ is equal to

(a) $1+x+x^2+2x^3$ (b) $1+x^2+4x^6$

(c) $1+7x^2+6x^4+x^6$ (d) $1+x^2+4x^4+4x^6$

Soln. $f(x)=x+2x^3,\ g(x)=1+x^2$

$h(x)=gof(x)=g(f(x))=1+(f(x))^2$

$h(x)=1+(x+2x^3)^2=1+x^2+4x^6+4x^4$

$h(x)=1+x^2+4x^4+4x^6$

Correct option is (d)

Ex. If $\phi(x)=x^2$ and $\psi(x)=2^x$ then $\psi(\phi(x))$ is

(a) 2^{x^2} (b) x^2 (c) x^{2x} (d) x^{2x}

Soln. $\phi(x)=x^2,\ \psi(x)=2^x$

$\psi(\phi(x))=2^{\phi(x)}=2^{x^2}$

Correct option is (a)

INVERTIBLE FUNCTION

A function $f:A\to B$ is invertible if there exists another function $g:B\to A$ such that $fog=I_B$ and $gof=I_A$, That is, $fog(x)=x\ \forall x\in B$ and $gof(x)=x\ \forall x\in A$. Then we say "g is inverse of function *f*".

(a) A function $f:A\to B$ is invertible if and only if *f* is bijective function.

(b) If $f:A\to B$ and $g:B\to C$ be two invertible function then $(gof)^{-1}=f^{-1}og^{-1}$.

Ex. Given $f(x)=x^2-1$ then $f^{-1}(x)$ has a domain given by

(a) $(-1,\infty)$ (b) $[0,\infty)$ (c) $(-\infty,\infty)$ (d) $[-1,\infty)$

Soln. Range of $f=B=[-1,\infty)$ because $x^2-1\ge -1\ \forall\ x\in\mathbb{R}$

Domain of f^{-1} = Range of $f=[-1,\infty)$

Ex. The function *f* is defined as $f(x)=ax+b$. The function *g* such that $g(f(x))=f(g(x))=x$ is given by

(a) $g(x)=\dfrac{x-b}{a}$ (b) $g(x)=\dfrac{x-a}{b}$ (c) $g(x)=\dfrac{x+b}{a}$ (d) $g(x)=\dfrac{x+a}{b}$

Soln. $g(f(x)) = f(g(x)) = x$

$\Rightarrow g(x) = f^{-1}(x)$

So, Let $f^{-1}(x) = t$

$\Rightarrow f(t) = x \Rightarrow at + b = x$

$\Rightarrow t = \dfrac{x-b}{a} \Rightarrow f^{-1}(x) = \dfrac{x-b}{a} \Rightarrow g(x) = \dfrac{x-b}{a}$

Correct option is (a)

Ex. If $f:\mathbb{R}\to\mathbb{R}$ is a mapping defined by $f(x) = x^2 + 1$ then $f^{-1}(17)$ and $f^{-1}(-3)$ have values respectively are

(a) $\phi, \{4,-4\}$ (b) $\{3,-3\}$ (c) $\phi, \{3,-3\}$ (d) $\{4,-4\}, \phi$

Soln. Let $f^{-1}(17) = t \Rightarrow f(t) = 17 \Rightarrow t^2 + 1 = 17 \Rightarrow t^2 = 16 \Rightarrow t = \pm 4$

$f^{-1}(17) = \{4,-4\}$

Again

$f^{-1}(-3) = x \Rightarrow f(x) = -3 \Rightarrow x^2 + 1 = -3 \Rightarrow x^2 = -4$

No real x exists

$f^{-1}(-3) = \phi$

Correct option is (d)

Ex. Let $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ be to maps such that *fog* and *gof* are continuous. Then

(a) f and g are continuous (b) $fog = gof$

(c) at least one of f or g is continuous (d) Neither f nor g may be continuous

Soln. If *fog* and *gof* are continuous then f and g are both continuous.

Correct option is (a)

Binary Operation

A function $*: A\times A \to A$ is called a binary operation on set A. We denote by $a*b$ or for a,b∈ A i.e., An operaton $*$ on set A is called a binary operation if for $a,b\in A \Rightarrow a*b\in A$.

Properties of Binary operation:

(i) Commutative : $a*b = b*a \quad \forall a,b\in A$

(ii) Associative : $(a*b)*c = a*(b*c)\ \forall a,b,c\in A$

(iii) Existence of Identity : There exists $e\in A$ such that $a*e\in a \quad \forall a\in A$.

The element e is called identity of $*$ in A. Identity of $*$ (if exists) is unique.

(iv) Existence of Inverse : For $a\in A$, there exists $b\in A$ such that $a*b = b*a = e$ where e is identity element of $*$. The element b is called inverse of a. Inverse of an element (if exists) is unique.

Ex. Consider the binary operations $*:\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ and o:$\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ defined as $a*b = |a-b|$ and $a\circ b = a$ for all $a,b\in\mathbb{R}$. which of the following is not true?

(a) $*$ is commutative but not associative (b) o is associative but not commutative

(c) $*$ is distributive over o (d) o is distributive over $*$

Soln. $a*b=|a-b|$

$a\circ b=a$

$a*b=b*a$ because $|a-b|=|b-a|$

$*$ is commutative.

$a\circ b=a$ but $b\circ a=b$

$a\circ b\neq b\circ a$ for some $a,b\in\mathbb{R}$

o is not commutative.

$a*(b*c)=a*|b-c|=|a-|b-c||$

$(a*b)*c=|a-b|*c=||a-b|-c|$

$a*(b*c)\neq(a*b)*c$ for some $a,b,c\in\mathbb{R}$

$\therefore$ '$*$' is not associative

$a\circ(b\circ c)=a\circ b=a$

$(a\circ b)\circ c=a\circ c=a$

$a\circ(b\circ c)=(a\circ b)\circ c\quad\forall a,b,c\in\mathbb{R}$

o is associative.

$a\circ(b*c)=a\circ(|b-c|)=a$

$(a\circ b)*(a\circ c)=a*a=|a-a|=0$

o is not distributive over $*$

$a*(b\circ c)=a*b=|a-b|$

$(a*b)\circ(a*c)=|a-b|\circ|a-c|=|a-b|$

$*$ is distributive over o

Correct option is (d)

Ex. Define $*$ *on* $\mathbb{Z}$ by $a*b=\frac{1}{2}\left[(a+b)+\frac{1}{2}\{1+(-1)^{a+b}\}+1\right]$ then $*$ is

(a) not a binary operation on $\mathbb{Z}$

(b) a commutative and associative binary operation on $\mathbb{Z}$

(c) a commutative binary operation on $\mathbb{Z}$ but not associative

(d) a binary operation on $\mathbb{Z}$ but neither commutative nor associative

Remark : A binary operation $*$ is distributive over 0 if $a*(b\circ c)=(a*b)\circ(a*c)$

Soln. Define $*$ *on* $\mathbb{Z}$ by $a*b=\frac{1}{2}\left[(a+b)+\frac{1}{2}\{1+(-1)^{a+b}\}+1\right]$

Let $a,b\in\mathbb{Z}$

Case (i) $a+b=$ even

$a*b=\frac{1}{2}[(a+b)+2]$ is also an integer

Case (ii) $a+b=$ odd

$a*b=\frac{1}{2}[(a+b)+1]$ is also an integer

Then $a*b \in \mathbb{Z}\ \forall a,b \in \mathbb{Z}$

$*$ is a binary operation.

$$b*a = \frac{1}{2}\left[(b+a)+\frac{1}{2}\left\{1+(-1)^{b+a}\right\}+1\right]$$

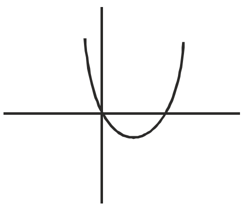
$a*b = b*a \quad \forall a,b \in \mathbb{Z}$

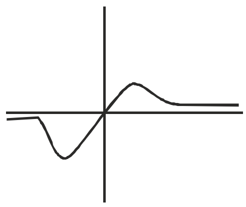
$*$ is commutative.

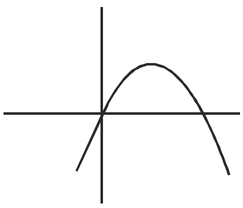
$(a*b)*c = a*(b*c) \quad \forall a,b,c \in \mathbb{R}$ $*$ is associative.

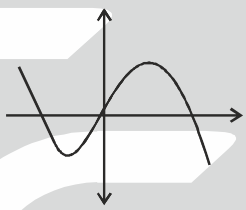
Correct option is (b)

Ex. The function $y = x(1-x)$ is best represented by

(a) 

(b) 

(c) 

(d) 

Soln. $y = 0 \Rightarrow x(1-x) = 0 \Rightarrow x = 0, 1$

option (b) & (d) are in correct

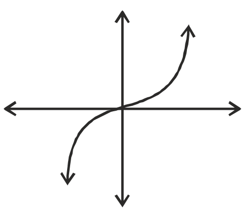
$y = x(1-x) = x - x^2$

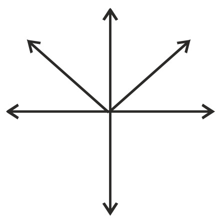
$f(0) = 0, f(1) = 0 \Rightarrow f\left(\frac{1}{2}\right) = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4} > 0$

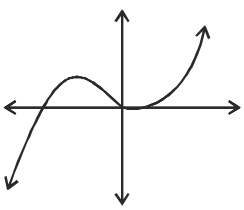
option (a) is incorrect

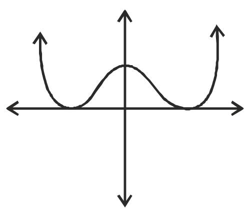
Correct option is (c)

Ex. Which of the following is most likely graph of the function $y = Ax^2 + Bx^4$ where $A > 0, B > 0$

(a) 

(b) 

(c) 

(d) 

Soln. Since $y = Ax^2 + Bx^4 \geq 0 \ \forall x \in \mathbb{R}$

option (a) & (c) are incorrect

option (b) is graph of $y(x) = |x| \Rightarrow$ option (b) is incorrect

$y(0) = 0 \Rightarrow$ option (d) is incorrect

None of the given options is correct

Infinite sets : A set containing infinite number of elements is called infinite set.

For example : $\mathbb{N}$, $\mathbb{W}$, $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{Q}'$, $\not\subset$ are infinite sets where

$\mathbb{N}$ = Set of natural numbers.

$\mathbb{W}$ = Set of whole numbers.

$\mathbb{Z}$ = Set of Integer numbers.

$\mathbb{Q}$ = Set of rational numbers.

$\mathbb{Q}'$ = Set of Irrational numbers.

$\mathbb{R}$ = Set of real numbers.

$\not\subset$ = Set of complex numbers.

Infinite set can be catogarised into two parts

(a) Countably infinite set

(b) Uncountable set

Countably infinite sets are $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{W}$ uncountable sets are $\mathbb{Q}'$, $\mathbb{R}$, $\mathbb{C}$

Cardinality of sets : In simple words, cardinality means number of elements present in the set. It is denoted by Card or | |.

<u>Sets with equal cardinality</u> :

Two sets A and B have the same cardinality if there exists a bijection $f : A \to B$. Then we write $|A| = |B|$.

For example, $\mathbb{N}$ and $\mathbb{Z}$ have same cardinality because $f : \mathbb{N} \to \mathbb{Z}$ defined by

$$f(n) = \begin{cases} \dfrac{n}{2} & \text{when } n \text{ is even} \\ -\dfrac{n-1}{2} & \text{when } n \text{ is odd} \end{cases}$$

is a bijective function.

Remark: The set $\mathbb{R}$ and Q' have same cardinality. In general, all countably infinite sets are of equal cardinality and all uncountable sets have same cardinality.

Ex. Let P be the set of all polynomials with rational coefficients. Then

(a) P is countable set
(b) P is an uncountable set
(c) P is a finite set
(d) P is the empty set

Soln. $P = \{a_0 + a_1 x + ... + a_n x^n \mid a_i \in Q \text{ for all } 0 \leq i \leq n\}$

Since rational numbers are countable and finite product of countable sets is countable.

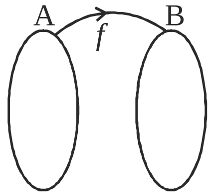
Therefore, P is countable set

Correct option is (a)

Ex. Which of the following statements about surjective map $f : A \to B$ is incorrect.

(a) If B is uncountable then A is also uncountable
(b) If A is infinite then B is also infinite
(c) If B is infinite then A cannot be finite
(d) Set B may be finite even if A is infinite

Soln. $f : A \to B$ is onto (surjective)



Every elements of B has at least one preimage in set A.

$\Rightarrow$ If B is uncountable then A is uncountable.

$\Rightarrow$ If B is infinite then A is also infinite.

Statement of (b) is incorrect, because if is constant function then $A = \mathbb{R}$ but $B = \{k\}$

Correct option is (b)

Ex. If $\mathbb{N}$ is set of natural numbers, $\mathbb{Z}$ is set of integers, $\mathbb{Q}$ is set of rational numbers, $\mathbb{R}$ is set of real numbers and $\mathbb{C}$ is set of complex numbers. There is a bijection between

(a) $\mathbb{N}$ and $\mathbb{Z}$ (b) $\mathbb{Z}$ and $\mathbb{R}$ (c) $\mathbb{R}$ and $\mathbb{Q}$ (c) $\mathbb{Q}$ and $\mathbb{C}$

Soln. $\mathbb{N}$ and $\mathbb{Z}$ are both countably infinite set $\Rightarrow$ we have a bijection between $\mathbb{Z}$ and $\mathbb{N}$

Correct option is (a)

Ex. Consider a one-one real valued function f defined on set of real number $\mathbb{R}$. Suppose a line ℓ parallel to x-axis intersect the graph G_f of f where

$G_f = \{(x, f(x) \mid x \in \mathbb{R}\}$. Then $G_f \cap \ell$ consists of

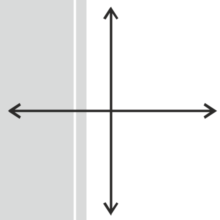
(a) infinitely may point (b) finitely man/y points

(c) Exactly two points (d) Exactly one point

Soln. $f : \mathbb{R} \to \mathbb{R}$ is one-one function

$\Rightarrow$ distinct elements of X has distinct images.

$\Rightarrow$ when we draw a line parallel to x-axis it will intersect graph at exactly one point.



Correct option is (d)

Ex. If $2x - y = 5,$ what is the value of $\frac{9^x}{3^y}$? **[JNU-2018]**

(a) 3^5 (b) 9^5 (c) 3 (d) 9

Soln. $2x - y = 5 \Rightarrow y = 2x - 5$

$$\therefore \frac{9^x}{3^y} = \frac{3^{2x}}{3^y} \quad (\because y = 2x - 5)$$

$$= \frac{3^{2x}}{3^{2x-5}} = 3^{2x} \cdot 3^{-(2x-5)} = 3^{2x-2x+5} = 3^5$$

Correct option is (a)

Ex. Given $\log_{10} 100 = \log_{10} 10^2 = 2$, What is the value of $\log_2 64$? **[JNU-2011]**

(a) 6.0 (b) 2.3 (c) 1.5 (d) 4.0

Soln. $\log_2 64 = \log_2 2^6 = 6 \log_2 2 = 6$ $(\because \log_a a = 1)$

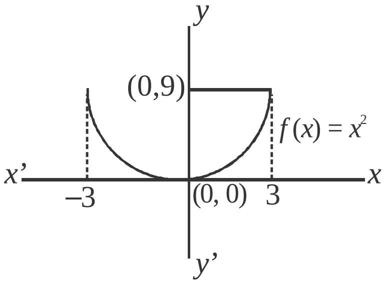
Correct option is (a)

Ex. For the function $f : x \to x^2$ with domain $x : -3 \le x \le 3,$ what is the range ? **[JNU-2011]**

(a) $\{y : 0 \le y \le 9\}$ (b) The set of all real numbers

(c) $\{y : -9 \le y \le 9\}$ (d) $\{y : y \le 3\}$

Soln.



So, range = [0, 9] = $\{y : 0 \le y \le 9\}$,

Correct option is (a)

Ex. Which of the following expression is /are True ? **[JNU-2011]**

$(\mathbf{E_1}); \dfrac{x^2 - y^2}{x + y} = x - y$

$(\mathbf{E_2}); \left(\sqrt{a} + \sqrt{b}\right)^2 = a + b$

(a) E_1 & E_2 (b) E_1 only (c) E_2 only (d) Neither E_1 nor E_2

Soln. $(E_1)\dfrac{x^2 - y^2}{x + y} = \dfrac{(x+y)(x-y)}{(x+y)} = (x - y)$

So, E_1 is True

$(E_2)\left(\sqrt{a} + \sqrt{b}\right)^2 = \left(\sqrt{a}\right)^2 + \left(\sqrt{b}\right)^2 + 2\sqrt{a}\sqrt{b} = a + b + 2\sqrt{ab}$

So, $(E_2)$is false

Correct option is (b)

Ex. If $\log_x 9 = 2,$ then $x =$ **[JNU-2011]**

(a) 4.5 (b) 18 (c) 3 (d) 2

Soln. $\log_x 9 = 2$ $\left(\begin{aligned} \therefore \log_a x &= \alpha \\ x &= a^\alpha \end{aligned}\right)$

$9 = x^2$

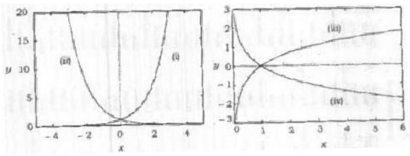
$3^2 = x^2$

Compairing both sides, we get

$\boxed{x = 3}$

Correct option is (c)

Ex. Choose the correct option with respect to figures given below : **[JNU-2015]**



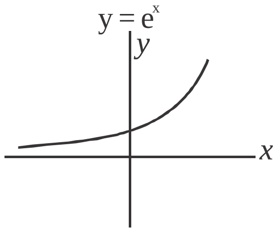
(a) (i) $\to e^x$, (ii) $\to e^{-x}$, (iii) $\to \log(1/x)$, (iv) $\to \log(x)$

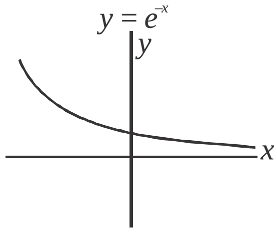
(b) (i) $\to e^{-x}$, (ii) $\to e^x$, (iii) $\to \log(1/x)$, (iv) $\to \log(x)$

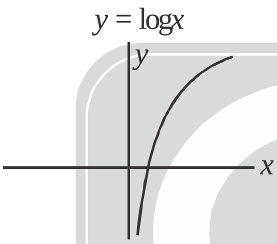
(c) (i) $\to e^x$, (ii) $\to e^{-x}$, (iii) $\to \log(x)$, (iv) $\to \log(1/x)$

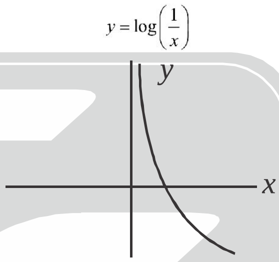
(d) (i) $\to e^{-x}$, (ii) $\to e^x$, (iii) $\to \log(x)$, (iv) $\to \log(1/x)$

Soln.









Correct option is (c)